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## Technical Note

1965-8

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Procedures and Computer Programs  
for Reduction of Ballistic Plates  
from Trailblazer I and II Programs

10 March 1965

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts



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MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
LINCOLN LABORATORY

PROCEDURES AND COMPUTER PROGRAMS FOR REDUCTION  
OF BALLISTIC PLATES FROM TRAILBLAZER I AND II PROGRAMS

F. A. WILSON

Group 21

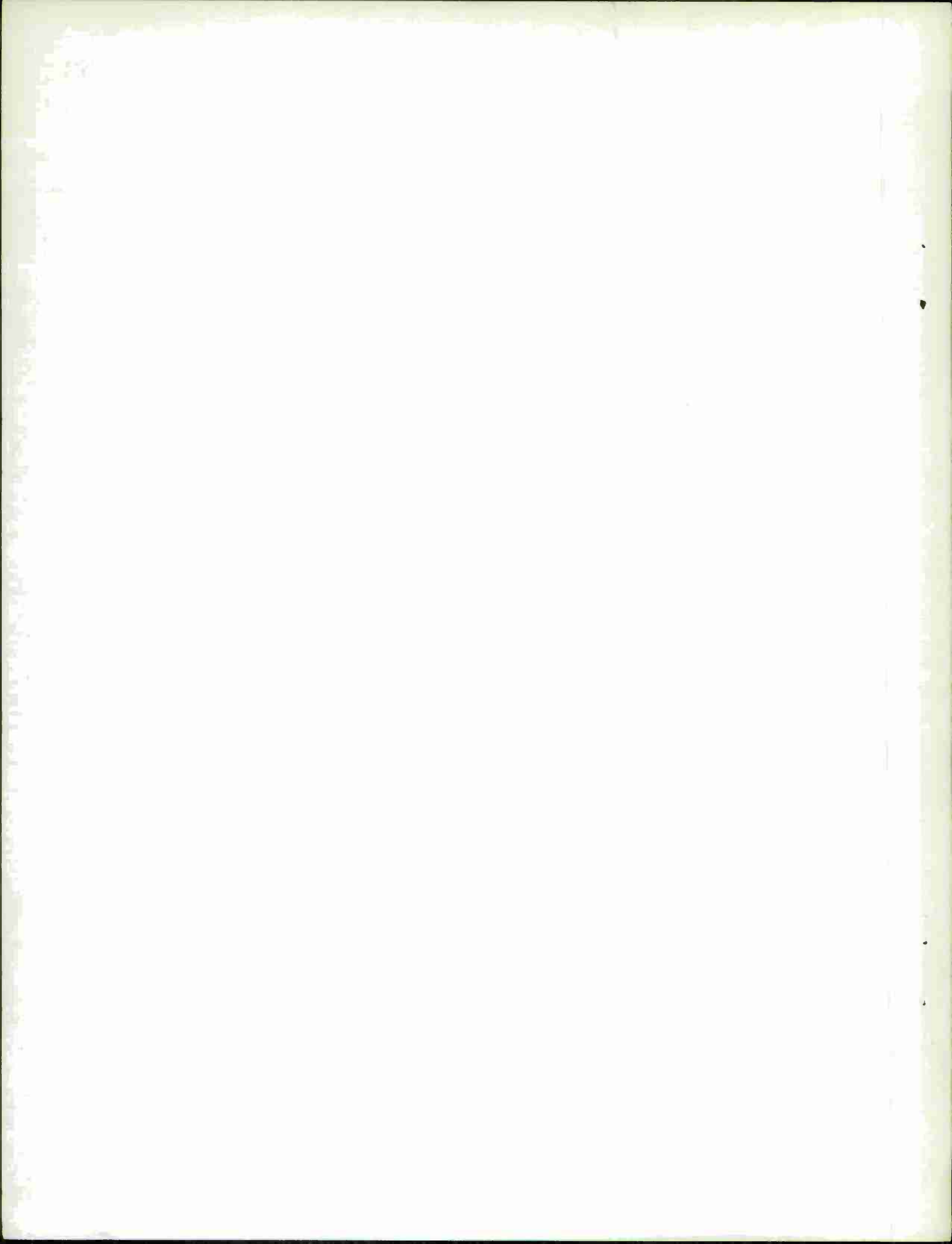
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TECHNICAL NOTE 1965-8

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LEXINGTON

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## ABSTRACT

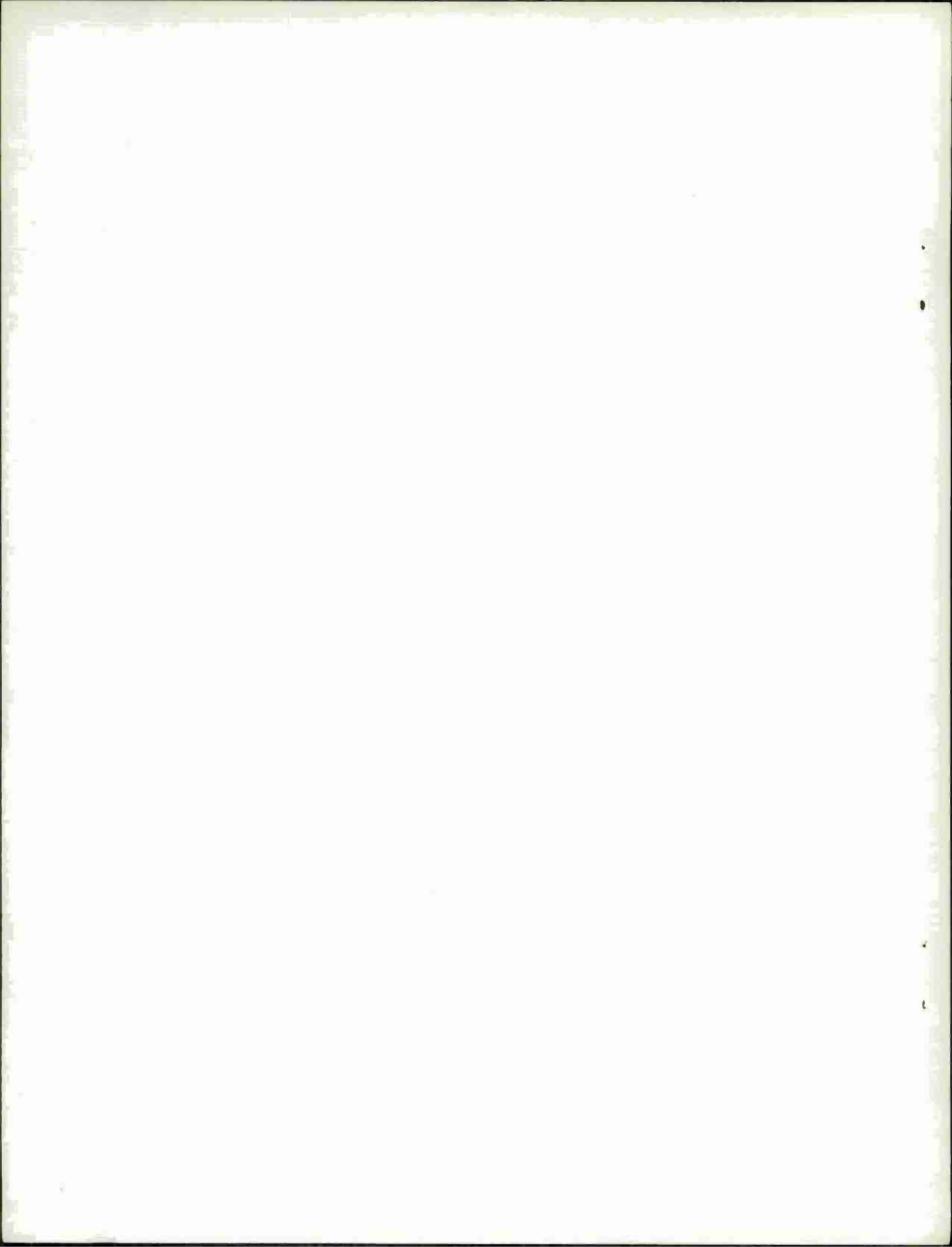
The Optical Reduction Programs have been written to obtain the space trajectory of a re-entering body from measurements of position on two photographic plates taken of a re-entry event from two separate stations. The method of analysis follows closely that developed by Whipple and Jacchia<sup>1</sup>. The Programs are a modification for the Lincoln Laboratory 7094 computer of a program originally written at the Harvard Observatory for reduction of meteor trails. The computation is divided into two programs:

1. The Plate Calibration Program calibrates the plate by using coordinates and plate measurements of known stars as reference points.

2. The Optical Trajectory Program computes range, height, time, direction cosines, and distance along the trail from the measured points on the trail.

This paper discusses these two programs in detail, including program listings, flow charts, and directions for running the programs. The mathematical background and the experimental method for obtaining the input data are also discussed.

Accepted for the Air Force  
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Lt Colonel, USAF  
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## GLOSSARY

### Sky and Plate Coordinates

Symbol	Definition
$a, A$	Altitude and azimuth as referred to the local horizontal
$\delta, H$	Declination and hour angle as referred to the equator
$\delta, \alpha$	Declination and right ascension as referred to the equator
$\beta, \lambda$	Celestial latitude and longitude as referred to the ecliptic
$\xi, \eta, \zeta$	Direction cosines in the altitude-azimuth system
$l, m, n$	Direction cosines in the declination-hour angle system
$\lambda, \mu, \nu$	Direction cosines in the declination-right ascension system
$\tau$	Vernal equinox or first point of Aries
$\epsilon$	Obliquity of the ecliptic ( $23\frac{1}{2}^\circ$ )
$\bar{\xi}, \bar{\eta}$	Standard rectangular plate coordinates with origin at the optical plate center
$X, Y$	Measured rectilinear coordinates on the photographic plate
$\sigma$	Angular distance from the optical plate center
$\Delta X, \Delta Y$	Corrections to measured rectilinear coordinates on the photographic plate
$X_x, X_y, X_z$ $Y_x, Y_y, Y_z$ $Z_x, Z_y, Z_z$	Precession direction cosines for the declination - right ascension system

$u_\alpha, u_\delta$	Proper motion in right ascension and declination per year
$a, b, c, d$	Plate constants in four constant solution
$A, B, C, D$	Inverse plate constants in four constant solution
$a_x, a_y, a_z, b_x, b_y, b_z$	Plate constants in six constant solution
$a_\xi, a_\eta, b_\xi, b_\eta, c_\xi, c_\eta$	Inverse plate constants in the six constant solution

#### Time and Terrestrial Coordinates

S.T. or $\theta$	Sidereal time at observer
$\theta_G$	Sidereal time at Greenwich
$\theta_0$	Sidereal time at 0000 Greenwich time at Greenwich
$\Delta\theta$	Correction to sidereal time required since a solar day is longer than a sidereal day
U.T.	Universal or Greenwich time
$L$	Terrestrial longitude
$\phi$	Terrestrial latitude (geographic)
$\phi'$	Terrestrial latitude (geocentric)
$\rho$	Radius of earth at sea level at observer's latitude
$R$	Radius of earth at observer
$\xi_{AB}, \eta_{AB}, \zeta_{AB}$	Direction cosines in equatorial system of the vector from station A to station B
$\xi_{AB_0}, \eta_{AB_0}, \zeta_{AB_0}$	Direction cosines in the equatorial system of the vector from station A to station B at sidereal time equal to zero at station A
$\lambda_z, \mu_z, \nu_z$	Direction cosines in the equatorial system of the zenithal point of the observer

## Other Symbols

$f$	Lens focal length
$l$	Length of the trail in angular measure
$Q$	Angle between the two meteor trails seen from the two stations
$S_A$	Distance from station A to the plane determined by the re-entry object and station B
$R_{Ai}$	Range from station A to the point $i$ on the re-entry object
$H$	Height above mean sea level
$D$	Distance along the trail from an arbitrary zero point
$P$	Period of revolution of shutter
$N$	Number of occultations of shutter per revolution
$t$	Relative time from arbitrary zero time
$\omega$	Angle that blade edge makes with dash

## Subscripts

$s$	Coordinates of stars
$c$	Coordinates of plate center
$q$	Center of rotating shutter

Double letter subscripts are used for points along the trail. The first letter is always A (for station A) or B (for station B)

$i$	A point on the trail - any shutter break or segment
$b$	Beginning of photographic trail
$e$	End of photographic trail



p	Pole of trail considered as a part of great circle
G	Greenwich
$\xi, \eta$	Refer to $\bar{\xi}$ and $\bar{\eta}$ standard plate axes
X, Y	Refer to X and Y axes on plate
R	Radiant of the event
Plate Calibration Program	

Computer Symbol	Math Symbol	Definition
TITLE	--	Title of event - any 72 characters desired
NN	--	Number of sets of precession constants
N	n	Number of stars
XX,XY,XZ	$X_x, X_y, X_z$	Precession constants
YX,YY,YZ	$Y_x, Y_y, Y_z$	
ZX,ZY,ZZ	$Z_x, Z_y, Z_z$	
ACHR,ACMIN,ACSEC	$\alpha_c$	Right ascension of plate center in hours, minutes, seconds
DCDEG,DCMIN,DCSEC	$\delta_c$	Declination of plate center in degrees, minutes, seconds
XC,YC	$X_c, Y_c$	Measured coordinates of plate center
M	--	Number of precession set to use with plate center or star
DTH	$\Delta\theta$	Correction in sidereal time to be used with non-tracking cameras
AC	$\alpha_c$	Right ascension of plate center in radians
DC	$\delta_c$	Declination of plate center in radians



ELC, EMC, ENC	$\lambda_c, \mu_c, \nu_c$	Direction cosines of unprecessed plate center in declination right ascension system
ELCP, EMCP, ENCP	$\lambda_c, \mu_c, \nu_c$	Direction cosines of precessed plate center in declination right ascension system
ELETA, EMETA, ENETA	$\lambda_\eta, \mu_\eta, \nu_\eta$	Direction cosines of $\bar{\eta}$ axis
ELEXI, EMEXI	$\lambda_\xi, \mu_\xi$	Direction cosines of $\bar{\xi}$ axis
AHR, AMIN, ASEC	$\alpha_s$	Right ascension of star in hours, minutes, seconds
A	$\alpha_s$	Right ascension of star in radians
DDEG, DMIN, DSEC	$\delta_s$	Declination of star in degrees, minutes, seconds
D	$\delta_s$	Declination of star in radians
X, Y	X, Y	Measured coordinates of star
ELS, EMS, ENS	$\lambda_s, \mu_s, \nu_s$	Unprecessed direction cosines of star in declination-right ascension system
EL, EM, EN	$\lambda_s, \mu_s, \nu_s$	Precessed direction cosines of star in declination-right ascension system
COSSIG	$\sigma_s$	Angular distance of star from plate center
EXI, ETA	$\bar{\xi}, \bar{\eta}$	Standard plate coordinates of star
A, B, C, D	a, b, c, d	Plate constants from four constant solution
ACAP, BCAP, CCAP, DCAP, ECAP	A, B, C, D, -A	Inverse plate constants from four constant solution
DX, DY	$\Delta X, \Delta Y$	Residuals in X and Y
CF, SF	$\cos \Delta\theta, \sin \Delta\theta$	Cosine and sine of $\Delta\theta$

ELC, EMC, ENC

ELEX, EMEX, ENEX

ELET, EMET, ENET

$\lambda_c', \mu_c', \nu_c'$

$\lambda_\xi', \mu_\xi', \nu_\xi'$

$\lambda_\eta', \mu_\eta', \nu_\eta'$

Corrected (for  $\Delta\theta$ ) direction cosines of the plate center and the standard axes in the declination-right ascension system

SIXCON

SEXI2

$\Sigma \bar{\xi}^2$

SEXIET

$\Sigma \bar{\xi} \bar{\eta}$

SEXI

$\Sigma \bar{\xi}$

SEXEXI

$\Sigma x \bar{\xi}$

SETA2

$\Sigma \bar{\eta}^2$

SETA

$\Sigma \bar{\eta}$

SEXETA

$\Sigma x \bar{\eta}$

SEX

$\Sigma x$

SEYEXI

$\Sigma y \bar{\xi}$

SEYETA

$\Sigma y \bar{\eta}$

SEY

$\Sigma y$

A

B

ASEXI, BSEXI, CSEXI

ASETA, BSETA, CSETA

$a_\xi, b_\xi, c_\xi$

$a_\eta, b_\eta, c_\eta$

Coefficient matrix

Inverse plate constants for six constant solution

# Optical Trajectory Program

TITLE	--	Title of event - any 72 characters desired
TITLE 1 (1)	--	Title of station A
TITLE 1 (2)	--	Title of station B
Constants		
CON	P/N	Period of revolution/number of occultations per revolution
DTRAD	$\pi/180$	Conversion of degrees to radians
F2	$f^2$	Focal length of camera squared
PI	$\pi$	3.141592654
Trail Equation		
EN,N	n	Number of points used to get trail equation
SLOPE	m	Slope of line found by trail equation
SUMX	$\Sigma X$	Sum of X readings
SUMX2	$\Sigma X^2$	Sum of X readings squared
SUMXY	$\Sigma XY$	Sum of the product of the X and Y readings
SUMY	$\Sigma Y$	Sum of the Y readings
XØ	$X_o$	First X reading on trail
YØ	$Y_o$	Average Y reading on trail

XTR	$\Delta X$	Difference between actual reading of point and $X_0$ ,
YTR	$\Delta Y$	$Y_0$ for points on trail
XBAR	$\bar{X}$	Average value of X reading
YBAR	$\bar{Y}$	Average value of Y reading
ZTRAIL	--	Y intercept of trail equation

#### Relative Coordinates

ELEV	--	Elevation of station above mean sea level
ELONH	--	Geographic longitude of station in hours, minutes, and seconds respectively
ELONM	--	
ELONS	--	
ELON	L	Geographic longitude of station in hours
ELZ, EMZ, ENZ	$\lambda_z, \mu_z, \nu_z$	Direction cosines of zenith of station
EXIABO, ETAABO, ZETAAB	$\xi_{AB_0}, \eta_{AB_0}, \zeta_{AB_0}$	Direction cosines of vector from station A to station B at sidereal time zero at station A and at the time of the event, respectively
EXIAB, ETAAB, ZETAAB	$\xi_{AB}, \eta_{AB}, \zeta_{AB}$	Direction cosines of vector from station A to station B at time of event
GTHR, GTM, GTS	--	Given event Greenwich time in hours, minutes, and seconds
PHD, PHM, PHS	--	Geographic latitude of station in degrees, minutes, and seconds
PHI	$\phi$	Geographic latitude of station in radians

CPHI, SPHI	$\cos \phi, \sin \phi$	Cosine and sine of latitude
PHIP, PHIPD	$\phi'$	Geocentric latitude of station in radians and degrees respectively
RAB	$R_{AB}$	Range from station A to station B
RHO	$\rho$	Earth's radius at the station
THET <del>PH</del> , THET <del>PM</del> , THET <del>PS</del>	$\theta_o$	Sidereal time at 0000 U.T. at Greenwich for the date of the event in hours, minutes, and seconds
DTHETS	$\Delta\theta$	Correction in sidereal time (seconds)
THET, THETA	$\theta$	Sidereal time in degrees or radians
CTH, STH	$\cos \theta, \sin \theta$	Cosine and sine of theta
UT	--	Universal time of event
DL	$\Delta L$	Difference in longitudes of the two stations
CDL	$\cos \Delta L$	Cosine and sine of $\Delta L$
SDL	$\sin \Delta L$	

#### Calculation of Pole and Radiant

CAX, CBX, CCX	$a_x, b_x, c_x$	Inverse plate constants for six constant solution (or four constant solution expanded to give six numbers)
CAY, CBY, CCY	$a_y, b_y, c_y$	
EXIBAR, ETABAR	$\bar{\xi}, \bar{\eta}$	Standard plate coordinates of a point
X	X	Measured reading of X on plate
Y	Y	Computed Y of point read at X

ELC, EMC, ENC	$\lambda_c, \mu_c, \nu_c$	Direction cosines of plate center declination-right ascension system
ELETA, EMETA, ENETA	$\lambda_\eta, \mu_\eta, \nu_\eta$	Direction cosines of $\bar{\eta}$ axis declination-right ascension system
ELEXI, EMEXI	$\lambda_\xi, \mu_\xi$	Direction cosines of $\bar{\xi}$ axis declination-right ascension system
EL, EM, EN	$\lambda, \mu, \nu$	Direction cosines of any point on trail declination-right ascension system
ELP, EMP, ENP	$\lambda_p, \mu_p, \nu_p$	Direction cosines of pole declination-right ascension system
DLONG	$l$	Trail length in degrees
SINL	$\sin l$	Sine of trail length
SINQ	$\sin Q$	Sine of angle of intersection between the two trails as seen in the sky from stations A and B
ELR, EMR, ENR	$\lambda_R, \mu_R, \nu_R$	Direction cosines of the radiant

#### Calculation of Ranges, Heights, and Distances

XP	$X_i$	X reading of any point
YP	$Y_i$	Corresponding Y computed for this X point
COSZ, COSZ1	$\cos Z$	Cosine of zenith distance of any point
D	$D$	Distance along the trail from an arbitrary zero
FEL, FEM, FEN	$\lambda, \mu, \nu$	Direction cosines of each point in declination-right ascension system

HEIGHT, H	H	Height above mean sea level for the point
DH	$\delta H$	Height correction
HL	h	Height above the tangent plane
RANGE, R	R	Range from the station to the point
NUM	--	Number of dashes
NDASH,ENDASH	--	Number of the dash (numbering from 1-NUM)
NWT	--	Weight or reliability of dash
EXIP,ETAP,ZETAP	--	Direction components of point declination-right ascension system

#### Calculation of relative time

XQ,YQ	$X_q, Y_q$	Observed coordinates of the center of rotation of the shutter
XC,YC	$X_c, Y_c$	Observed coordinates of the projection center
DX,DY	--	Differences in two centers
XMEAS	X	X reading of any point
DYTR	--	Difference between Y computed and Y of rotation center
OMEGA	$\omega$	Angle that blade makes with dash
OCCULT	N	Number of occultations per revolution
P	P	Period of revolution

SIGN, IF PLUS	--	Code describing direction of revolution of shutter
IFEXT	--	Code if cards are desired
ELAZ,EMAZ,ENAZ	$\xi, \eta, \zeta$	Direction cosines of point in the altitude-azimuth system



## I. INTRODUCTION

The optical reduction programs for the 7094 computer have been written to compute the space trajectory of the re-entering bodies of the Trail-blazer experiments fired from Wallops Island, Virginia. When a body re-enters the atmosphere it is heated and may become luminous; thus, it may be photographed. Trails are recorded photographically from several stations by Lincoln Laboratory and the NASA and the photographs are available for analysis. The programs discussed in this report compute positions along the re-entry trail using known stars as calibrations. Relative time may be introduced by the use of chopping shutters on one or more cameras, so that breaks at regular time intervals are produced along the trail.

The mathematical background of the reduction procedure is discussed in Section II and the experimental techniques in Section III. The programs themselves are documented in Sections IV and V. The astrometric reduction is modeled after the technique discussed by Whipple and Jacchia<sup>1</sup> and McCrosky<sup>2</sup>. The programs are adapted from programs written by Dr. McCrosky's group at the Harvard Observatory for meteor analysis.

A camera usually maps the celestial sphere onto a flat focal plane. In the case of the Super-Schmidt Cameras, the focal plane is curved and the film must be copied by projection onto a flat plate for analysis. All measurements for this reduction are made on the flat plates. Positions on the plate are measured as discussed in Section III.

The angular direction to the reference stars and the vehicle trail are determined with respect to each camera station, and from these directions the angular bearing of measured points along the trail can be found. The trails from the two stations may look quite different since they are photographed at different distances and angles. Also, clouds sometimes obscure parts of the trail. Thus the beginning and end points of the trails recorded from the two stations may not be identical. No attempt has been made to synchronize the chopping shutters in the Wallops Island camera systems.

Since the Trailblazer vehicles are axially symmetric, the only force acting to change the direction of the re-entry from a straight line is the gravity effect on a freely falling body. This effect is generally small over the short time of the event and the deviation from a straight line trajectory, if any, would be found near the end of the trail. Also the re-entry body of the Trailblazer system is fired downward and re-enters about  $10-12^\circ$  from the vertical so that gravity acts mainly along the vehicle trajectory. Thus the re-entry trail is usually a straight line on the plate. If the trail is curved, it must be broken up into straight line segments and the program must be run separately for each straight line part.

From the known positions of stars near the trail at the time of the event, a calibration is performed on each plate; from the results of this calibration, the angular position of points along the trail may be determined for each plate. Since the trail appears as a straight line, the position vectors of points along the trail as seen from the camera station must determine a unique plane. The re-entry must lie on the intersection of the two planes determined from a pair of photographs taken at separate stations. The distance from the camera site to the point on the line of intersection may be computed individually for each point. Therefore, a set of slant ranges  $R_1$  for points along the trail may be found for each plate. Height, direction cosines, and relative distance along the trail may then be computed for each measured point along the trail. If the plates were obtained from a camera with a chopping shutter, relative time may be computed for each dash.

In general a unique time for any given point on the plate can not be found. Also, no exact correlation exists between points on the two trails. If a common point such as a flare can be identified on both plates, the points may be considered to occur at the same time. The height of the point computed from both stations should be identical (this will serve as a check on the height computations). In this case, also, the exact

sidereal time of the flare can be computed; however, since in general flares need not occur, this computation of sidereal time is not included in the present program. In the Project PRESS coded shutters have been built into the cameras and the chopping shutters have been synchronized so that absolute time can be recorded and an analysis performed using unique time points from two or more stations. The Super-Schmidt Cameras used in the Trailblazer program are not equipped to record absolute time since suitable coded shutters can not be easily installed. It should be noted that the re-entry events of Project PRESS are much longer than those of the Trailblazer series; thus the design and use of absolute timing equipment presents much less difficulty in Project PRESS.

The present set of programs is designed to compute the position of the body, i.e. height, range, and direction cosines of each point along the trail. If the trail is chopped, relative times can be computed for the dashes; relative distances along the trail from an arbitrary point on the trail are also computed. These programs do not compute velocity. If the distance along the trail as a function of time is known, various methods of computing velocity might be used. Harvard has obtained good results in meteor work with the equation

$$D = a + bt + c \exp (kt)$$

where D is the distance along the trail; t is the elapsed time from an arbitrary reference point on the trail and a, b, c and k are computed constants. However, this fit did not appear particularly suitable for the Trailblazer work. At present velocity is computed from the results of the optical trajectory program by hand computation. Since this method is not yet suitable for machine calculation, the computation of velocity has been omitted from the present program.



## II. MATHEMATICAL BACKGROUND

### A. Astronomical Coordinate Systems

When the stars are observed from a point on earth they appear to lie on the surface of a sphere with the observer at its center. Although the eye can not determine the distance to the stars, an estimate of the angles subtended at the observer by any pair of stars can easily be made. These directions are defined in terms of the positions on the surface of a sphere - the celestial sphere - in which the straight lines joining the observer to the stars intersect this surface. The axis of the earth pierces the sphere at two points called "the celestial poles". The radius of the celestial sphere is considered arbitrarily large.

The position of any point on the surface of a sphere may be completely specified by reference to two principal great circles on the sphere, or by one great circle and a point on that great circle. Thus a point on the surface of the earth is completely specified by its latitude, measured from the equator, and its longitude, measured east or west from Greenwich. Several methods of defining position on the celestial sphere are in common use.

#### 1. Spherical Coordinate Systems

##### a. The Altitude-Azimuth System

Let  $O$ , the observer on the surface of the earth, be the center of the celestial sphere.  $Z$ , the zenith, is the point directly overhead (defined by a plumb line) and the nadir is the point directly underfoot. The plane through  $O$  perpendicular to the direction of the zenith is called the plane of the horizon and, as shown in Fig. 1a, cuts the celestial sphere in the great circle  $NAS$  dividing the celestial sphere into two hemispheres, the upper being visible and the lower being hidden by the earth itself. A vertical circle is a great circle passing through the zenith and the nadir.

Let  $X$  be the position of a star on the celestial sphere at a given moment. The angle  $\widehat{AOX}$  or the great circle arc  $AX$  is called the altitude,  $a$ , of the star. To completely specify the position of a point on the celestial sphere the particular vertical circle on which it lies must be specified. Let  $OP$  be parallel to the earth's axis. If the observer is in a northern latitude the position  $P$  is called the north celestial pole. The vertical circle through the pole,  $ZPN$  in Fig. 1a, is defined as the principal vertical; it is the circle traced on the celestial sphere by the observer's meridian. The point where this circle cuts the horizon is called the north point,  $N$ , of the horizon plane; the point  $S$  directly opposite  $N$  on the horizon is the south point.

If the star is in the eastern\* part of the sphere, as is  $X$  in Fig. 1a, the spherical angle  $\widehat{PZX}$  or the great circle arc  $NA$  is called the azimuth  $A$ ; if the star is in the western part of the sky, as is  $Y$ , the azimuth is the angle  $\widehat{PZY}$  which in this case is greater than  $180^\circ$ . Throughout this report azimuth will always be measured from north in an easterly direction from  $0-360^\circ$ . Since the angle  $\widehat{POZ}$  in Fig. 1a equals the angle between the radius of the earth which passes through the observer's position and the earth's axis,  $\widehat{POZ}$  is seen to be equal to the co-latitude of the observer.

#### b. The Declination-Hour Angle System

The celestial equator is the great circle of the celestial sphere perpendicular to the polar axis and is thus the great circle in which the plane of the earth's equator cuts the celestial sphere,  $RWT$  in Fig. 1b. The celestial equator and the horizon intersect at two points  $E$  and  $W$  which are each  $90^\circ$  from  $N$  and  $S$  and are called the east and west points.

Let  $X$  be a star in the northern hemisphere. If  $PXD$  is the semi-great circle through  $X$  and the pole  $P$ , the arc  $DX$  is defined as the declination  $\delta$  of  $X$ , i.e. its distance in degrees north (+) or south (-) of the celestial equator. A great circle passing through the pole of the celestial sphere

---

\*If the observer faces north, the west is on his left and the east on his right.

is called a meridian or an hour circle. The second great circle required for complete specification of the star's position in the declination-hour angle system is taken as the observer's meridian (i.e. that meridian passing through the zenith) PZRSQ (L) in Fig. 1b. At any moment the position of X is specified by the angle at the pole between the observer's meridian and the meridian PXQ through the star at that time. This angle  $\widehat{RPX}$  or  $\widehat{ZPX}$  or the arc RD is called the hour angle H and is measured from the observer's meridian westward  $0^\circ$  -  $360^\circ$  or 0h - 24h.

#### c. The Declination-Right Ascension System

In the hour angle-declination system only one coordinate, the declination, remains constant as the earth rotates whereas the hour angle increases uniformly from 0h - 24h during a day. It is usually more convenient to define the second coordinate relative to a fixed point on the equator (although all the stars appear to change position during the daily rotation of the earth, their relative positions with respect to one another remain unchanged over long periods of time). The point chosen as standard is the vernal equinox or the first point of Aries denoted by  $\gamma$ . Then the arc  $\gamma D$  or angle  $\widehat{\gamma PX}$  is called the right ascension of star X, denoted by  $\alpha$ , and is measured eastward from 0h - 24h (or from  $0^\circ$  -  $360^\circ$ ) Fig. 1b. Note: right ascension is measured in the opposite direction to hour angle.

#### d. The Celestial Latitude - Longitude System

The earth is a planet rotating around the sun in an elliptical orbit with the sun at one focus with a period of revolution of approximately one year. During the year, as observed from earth, the sun appears to make a complete circuit of the sky against the star background. The plane of this orbit is called the plane of the ecliptic and the great circle in which this plane cuts the celestial sphere is called the ecliptic. This plane is found to be inclined at an angle of about  $23\frac{1}{2}^\circ$  to the celestial



equator; this inclination is called the obliquity of the ecliptic, denoted by  $\epsilon$ ;  $\epsilon = \hat{M}^{\wedge}R$  in Fig. 1c.

Relative to the earth the sun appears to move on the surface of the celestial sphere along the ecliptic in the direction  $Y^{\wedge}M$ , and twice a year its position on the celestial sphere coincides with the intersection of the ecliptic with the celestial equator. This position  $\tau$  at which the sun's declination changes from south to north is defined as the vernal equinox; it is the reference point in the right ascension-declination system.

A fourth set of coordinates can be derived using the ecliptic as a fundamental great circle and the vernal equinox  $\tau$  as a principal reference point. In Fig. 1c let  $K$  be the pole of the ecliptic and  $KXA$  a great circle arc passing through star  $X$  meeting the ecliptic in  $A$ . The arc  $\tau A$  measured from  $\tau$  to  $A$  along the ecliptic in the direction of the sun's annual motion, i.e. eastward, is called its celestial longitude  $\lambda$  and is measured from  $0$  -  $360^\circ$ . The arc  $AX$  is called the celestial latitude  $\beta$  and is measured north or south of the ecliptic.

#### e. Notes on Coordinate Systems

In the previous definitions, the center of the celestial sphere was taken as the observer on the surface of the earth. However, since the stars are at distances which are almost infinitely large as compared with the dimensions of the earth, no appreciable error results from taking the center of the earth as the center of the celestial sphere; this convention will be used in this report when dealing with the star background. Although our figures have been drawn for the northern hemisphere, similar figures apply for the southern and the definitions of the coordinate systems apply to both hemispheres. For an observer exactly at the pole the definitions of azimuth and meridian break down; thus in the following study it will be assumed that the observer is not at a pole.

### 2. Rectangular Coordinate Systems

In the preceding discussion, points on the celestial sphere were defined in terms of spherical coordinates; however, in most of the following

work it is much more convenient to use a rectangular reference system. A right handed coordinate system is chosen in which the coordinates  $x, y, z$  are measured along three perpendicular axes such that the rotation of the  $x$  axis into the  $y$  axis around the  $z$  axis is accomplished by the right hand rule; the origin of the system is taken as the observer or the earth's center (see above). The position of a star on the celestial sphere may be represented in vector form by

$$\bar{r} = r_x \bar{i} + r_y \bar{j} + r_z \bar{k} \quad (1)$$

where  $\bar{i}, \bar{j},$  and  $\bar{k}$  are unit vectors in the  $x, y, z$  directions respectively, and  $r_x, r_y$  and  $r_z$  are the projections of  $\bar{r}$  on the respective axes. The cosines of the angles between the positive  $x, y,$  and  $z$  axes and the vector  $\bar{r}$  measured in the direction from the axis toward  $\bar{r}$  are called the direction cosines of  $\bar{r}$ . We will express all coordinates in terms of direction cosines; in this way the magnitude of  $\bar{r}$  will not appear and only the directions will be used.

#### a. The Altitude-Azimuth System

The horizon will be taken as the  $xy$  plane, the positive  $x$  axis will pass through the east point of the horizon, the positive  $y$  axis through the north point, and the positive  $z$  axis through the zenith.

Let  $S$  in Fig. 2a be any star on the celestial sphere with altitude  $a$  and azimuth  $A$ . Then using  $\xi, \eta, \zeta$  to represent direction cosines in the  $x, y, z$  directions, respectively

$$\begin{aligned} \xi &= \cos a \sin A \\ \eta &= \cos a \cos A \\ \zeta &= \sin a \end{aligned} \quad (2)$$

#### b. The Declination-Hour Angle System

The plane of the equator will be taken as the  $xy$  plane, the positive  $x$  axis will pass through the west point on the equator, the positive  $y$  axis through the intersection of the observer's meridian and the equator,



and the positive z axis through the celestial north pole. Thus, in the form of direction cosines  $l, m, n$  (Fig. 2b)

$$\begin{aligned} l &= \cos \delta \sin H \\ m &= \cos \delta \cos H \\ n &= \sin \delta. \end{aligned} \tag{3}$$

#### c. The Declination-Right Ascension System

The plane of the equator will be taken as the xy plane; the positive x axis will pass through the vernal equinox  $\tau$ , the positive y axis through the point whose right ascension is  $+90^\circ$ , and the positive z axis through the celestial north pole. Thus (Fig. 2c)

$$\begin{aligned} \lambda &= \cos \delta \cos \alpha \\ \mu &= \cos \delta \sin \alpha \\ \nu &= \sin \delta \end{aligned} \tag{4}$$

where  $\lambda, \mu, \nu$  are direction cosines in the x, y, z directions respectively.

#### B. Precession and Proper Motion

Although the phenomenon of precession was observed by Hipparchus over 2000 years ago, the first correct dynamical explanation was given by Newton. As is well known the earth has the shape of an oblate spheroid rather than that of a sphere due to an equatorial bulge produced by its rotation about its polar axis; this bulge is inclined to the plane of the ecliptic by about  $23 \frac{1}{2}^\circ$ . Thus the solar and lunar attractions produce moments about the center. Since the earth is spinning rapidly on its axis much like a top, instead of turning the equatorial plane into coincidence with the ecliptic plane, the result of these attractive forces is to cause the axis of the earth to precess about the axis of the ecliptic. Thus the earth's axis has a conical motion about the pole of the ecliptic so that the pole of the equator describes a small circle about the ecliptic pole and the vernal equinox moves backwards along the ecliptic.

The precession of the equinoxes causes the celestial longitude of a star to increase at the rate of about 50 seconds of arc per year while the celestial latitude shows no significant change. In our current study we are interested primarily in the effect of precession on the right ascension and declination of a star. When the coordinates of the star are expressed in direction cosine form precession may be considered the result of a rotation of the axes of the system with the origin unchanged. Let us consider the effect on a star S with direction cosines  $\lambda, \mu, \nu$  relative to the original axes X, Y, Z of a rotation of these axes to a new position X', Y', Z' where the angles between OX', OY', OZ' and OX, OY, OZ are shown in Table I where, for example,  $X_z$  represents the angle between OX' and OZ.

TABLE I

	OX	OY	OZ
OX'	$X_x$	$X_y$	$X_z$
OY'	$Y_x$	$Y_y$	$Y_z$
OZ'	$Z_x$	$Z_y$	$Z_z$

Thus the transformation is given by the matrix equation

$$\begin{pmatrix} \lambda' \\ \mu' \\ \nu' \end{pmatrix} = \begin{pmatrix} X_x & X_y & X_z \\ Y_x & Y_y & Y_z \\ Z_x & Z_y & Z_z \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix} \quad (5)$$

The direction cosines  $\lambda', \mu', \nu'$  giving the position of a star at any given time can be easily obtained from the direction cosines  $\lambda, \mu, \nu$  at a standard equinox if the values of the transformation matrix are known. Usually the right ascension and declination of a star are tabulated for the equinox of 1950 in star catalogues; the values of the transformation matrix can be found<sup>3</sup>.

In 1718 Halley discovered that the position of certain bright stars had changed appreciably in relation to the general star background since the time of Hipparchus. This suggested that these stars had a definite space velocity relative to the sun and lay at finite distances from the earth. The proper motion of a star is defined as its apparent angular rate of motion on the celestial sphere; it is usually measured in seconds of arc per year. The proper motion is usually found by comparing precise observations of the right ascension and declination of the star taken many years apart. The star catalogues give the components  $\mu_{\alpha}$ , the proper motion in right ascension, and  $\mu_{\delta}$ , the proper motion in declination, per year or per century for each star tabulated. Since the components of proper motion depend upon the epoch and equinox for which they are measured, it is important to apply proper motion corrections to the tabulated values of  $\alpha$  and  $\delta$  for a star before precession is applied. Thus, for example, suppose the right ascension and declination of star S in 1935 referred to the equinox of 1950.0 is known and it is desired to compute the coordinates of the star in 1964 referred to the 1964.0 equinox. The simplest method is to first apply proper motion corrections from the tabulated values to find the position of S in 1964 relative to the equinox of 1950.0 and then to precess the star to the equinox of 1964. A detailed discussion of precession and proper motion may be found in Smart<sup>4</sup>.

Another effect of some interest is nutation which is a slight periodic oscillation of the actual pole around the mean pole where the mean pole is precessing uniformly. Since the effects of this nutation are small no corrections for them appear in this work.

### C. Time

The sidereal time at any place on earth at any instant is, by definition, the hour angle of the vernal equinox at that instant. When  $\tau$  is on the

observer's meridian the sidereal time is 0; when  $\tau$  is next on the observer's meridian after 24h of sidereal time or one complete revolution of the earth about its axis, a sidereal day has passed. From Fig. 1b it is obvious that

$$R\tau = RD + \tau D$$

Now RD is the hour angle of star X and  $\alpha$  is its right ascension; moreover  $R\tau$  is the hour angle of  $\tau$ . Therefore

$$\begin{aligned} \text{Sidereal time (observer)} &= \text{hour angle X} + \text{right ascension X} \\ \text{S.T.} &= H + \alpha \end{aligned} \quad (6)$$

Now let G on the celestial sphere in Fig. 1b be the zenith of Greenwich;  $\widehat{GPZ}$  is the longitude of the observer,  $\widehat{GPX}$  the hour angle of X from Greenwich and  $\widehat{ZPX}$  the hour angle of X from the observer's meridian  $\angle$ . Since

$$\widehat{GPX} = \widehat{ZPX} - \widehat{ZPG} = \widehat{ZPX} - \text{longitude of observer}$$

Obviously:

$$\text{H.A. of X at Greenwich} = \text{H.A. of X at } \angle \pm \text{longitude of } \angle$$

Since this relation is general it also holds for  $\tau$ . Thus

$$\text{S.T. at Greenwich} = \text{S.T. at } \angle \pm \text{longitude at } \angle \quad (7)$$

The sign of  $\angle$  is + when  $\angle$  is west of Greenwich and - when  $\angle$  is east of Greenwich.

A mean solar day is defined as the interval between two successive transits of a mean sun\* across the observer's meridian and is the day used in our system of civil time. A sidereal day has been found to be 3m 56.556s shorter than a mean solar day.

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\*The mean sun is a fictitious body which is defined to move in the celestial equator at a uniform rate around the earth so that it completes a revolution in the same time as the true sun completes a circuit of the ecliptic.



In our later work it will often be necessary to compute the sidereal time at a point on earth at a given local time. It is most convenient to compute the sidereal time at Greenwich first and then to use equation 7 to find local sidereal time. The American Ephemeris and Nautical Almanac<sup>5</sup> for the proper year contains a table called "Universal and Sidereal Times" which gives the sidereal time for each day of the year for 0h Greenwich time for that day, i.e., the sidereal time at midnight at Greenwich for the given day; each day starts at a different sidereal time since the length of a day in the two systems is different. The given local time is changed into Universal time by adding the proper correction (+5 hours for E.S.T.). The sidereal time at Greenwich is next calculated by

$$\theta_G = \theta_0 + \text{U.T.} + \Delta\theta \quad (8)$$

where  $\theta_G$  is the sidereal time at Greenwich,  $\theta_0$  is the sidereal time at 0h UT taken from Ref. 5 and  $\Delta\theta$  is the correction required since a solar day is longer than a sidereal day.

$$\Delta\theta = 3\text{m } 56.556\text{s (U.T.h/24h)} = 236.556 \text{ (U.T.h/24h) sec} \quad (9)$$

To find the sidereal time at station A from its value at Greenwich at the same time we merely use equation 7 and the whole equation then becomes

$$\theta_A = \theta_0 + \text{UT} - L_A + \Delta\theta \quad (10)$$

where  $\theta_A$  is the desired time at A and  $L_A$  is the longitude of A (west in this case).

#### D. Transformation of Coordinates

In our later work it will be necessary to transform between the altitude - azimuth and the declination - right ascension systems. Let us consider the problem of converting from the right ascension-declination system to the altitude-azimuth system in direction cosine form. We are given  $\alpha$  and  $\delta$  for star X at a given time  $t$  and a given latitude  $\phi$ . We must first transform our three direction cosines  $\lambda, \mu, \nu$ , to the  $l, m, n$  cosines

of the declination-hour angle system (Fig. 3a).

It is obvious that  $n = \nu$  since the axis is identical for the two systems and the  $x, y$  plane is the same in both cases.

$\theta = \alpha + H$  where  $\theta$  is sidereal time at the observer

so that the transformation may be seen to take place as in Fig. 3b where the rotation of axes in the  $x, y$  plane is shown.

$$\begin{aligned} l &= \lambda \cos(90 - \theta) - \mu \sin(90 - \theta) = \lambda \sin \theta - \mu \cos \theta \\ m &= \lambda \sin(90 - \theta) + \mu \cos(90 - \theta) = \lambda \cos \theta + \mu \sin \theta \\ n &= \nu \end{aligned} \quad (11)$$

Next, we must transform from the declination-hour angle system to the altitude-azimuth system (Fig. 3c). The  $y, z$  -plane is common to both systems and  $x_3 = -x_2$ , so that immediately

$$\xi = \mu \cos \theta - \lambda \sin \theta$$

Also,  $PZ = 90 - \phi$  so that the transformation may be represented in the plane as in Fig. 3d.

$$\begin{aligned} \xi &= \mu \cos \theta - \lambda \sin \theta \\ \eta &= -[m \cos(90 - \phi) - n \sin(90 - \phi)] = \nu \cos \phi - (\lambda \cos \theta + \mu \sin \theta) \sin \phi \\ \zeta &= m \sin(90 - \phi) + n \cos(90 - \phi) = (\lambda \cos \theta + \mu \sin \theta) \cos \phi + \nu \sin \phi \end{aligned} \quad (12)$$

#### E. Astronomical Photography

A section of the celestial sphere can be photographed by using the general configuration of Fig. 4. Let CO be the optical axis of a lens with focal length  $f$  and assume that CO passes through the geometric center of the plate. The focal plane of the lens, FG, is perpendicular to OC. Let us consider a star on the optical axis at C; all the rays from this star falling on the lens will be brought to a focus at O and an image formed at that point. If another star lies on the celestial sphere at S, it is seen to form an image at R. If the astronomical coordinates of C and S are known, since  $f$  is the known focal length of the lens and  $\sigma = b$ , we can find the relation between the linear distance  $d$  on the photographic plate and the corresponding

angular distance  $\sigma$  in the sky. Thus

$$d = f \tan b = f \tan \sigma \quad (13)$$

We will now define a system of axes on the plate through the plate center such that the position of a star on the plate can be related to its astronomical coordinates. First, the direction cosines for the plate center in the right ascension-declination system are found by applying equation 4 to the special case of the plate center.

$$\begin{aligned} \lambda_c &= \cos \delta_c \cos \alpha_c \\ \mu_c &= \cos \delta_c \sin \alpha_c \\ \nu_c &= \sin \delta_c \end{aligned} \quad (14)$$

Standard coordinates  $\bar{\xi}$  and  $\bar{\eta}$  will be rectangular coordinates in the plane tangent to the celestial sphere at C with their origin at C such that  $\bar{\eta}$  increases northerly along the hour circle through C and  $\bar{\xi}$  increases toward the east tangent to the parallel of declination of C (Fig. 5).

Having found the direction cosines of the plate center in terms of the declination right-ascension system, it is now desired to determine the direction cosines of the plate axes  $\bar{\xi}$  and  $\bar{\eta}$ , defined above, using the same system. The angle OCP' is a right angle since  $\bar{\eta}$  on the sphere must be perpendicular to a radius OC of the sphere. Therefore, OP'C must equal  $\delta_c$  and so  $\nu_{\bar{\eta}} = \cos \delta_c$ . Also the projection of  $\bar{\eta}$  on the equatorial plane must be  $-\sin \delta_c$  so that the direction cosines of  $\bar{\eta}$  are given by

$$\begin{aligned} \lambda_{\bar{\eta}} &= -\sin \delta_c \cos \alpha_c \\ \mu_{\bar{\eta}} &= -\sin \delta_c \sin \alpha_c \\ \nu_{\bar{\eta}} &= \cos \delta_c \end{aligned} \quad (15)$$

Since  $\bar{\xi}$  increases along a tangent to a parallel of declination, its projection on the z axis is zero so that  $\nu_{\bar{\xi}} = 0$ . The  $\bar{\xi}$  axis is directed eastward from the point on the sphere representing the plate center; hence its direction is  $\alpha_c + 90^\circ$  (Fig. 5). Hence:

$$\begin{aligned}
\lambda_{\xi} &= - \sin \alpha_c \\
\mu_{\xi} &= + \cos \alpha_c \\
\nu_{\xi} &= 0
\end{aligned}
\tag{16}$$

Now let S be a star on the celestial sphere with coordinates  $(\alpha_s, \delta_s)$  and direction cosines in our normal equatorial system  $(\lambda_s, \mu_s, \nu_s)$ . Let us find the direction cosines of S relative to the coordinate system of the photographic plate  $(\bar{\xi}, \bar{\eta})$ . The direction cosines of the standard plate system's axes relative to the equatorial axes are shown in Fig. 6b. Thus the direction cosines of S in the plate-centered system are

$$\begin{aligned}
P_{\bar{\xi}} &= \lambda_{\xi} \lambda_s + \mu_{\xi} \mu_s \\
P_{\bar{\eta}} &= \lambda_{\eta} \lambda_s + \mu_{\eta} \mu_s + \nu_{\eta} \nu_s \\
P_z &= \lambda_c \lambda_s + \mu_c \mu_s + \nu_c \nu_s
\end{aligned}
\tag{17}$$

It is also true that

$$\cos \sigma_s = \lambda_c \lambda_s + \mu_c \mu_s + \nu_c \nu_s
\tag{18}$$

where  $\sigma_s$  is the angle between the star and the plate center. From Fig. 4 it can be seen that the star located on the celestial sphere at S is seen by the camera at R, as if it were located on the tangent plane at D. This type of projection is called a gnomonic projection and it is assumed that the camera lens is located at the center of the celestial sphere. The gnomonic projection is accomplished by dividing equation 17 by the cosine of  $\sigma_s$ . One then obtains the pair of standard plate equations for the star S:

$$\begin{aligned}
\bar{\xi}_s &= (\lambda_{\xi} \lambda_s + \mu_{\xi} \mu_s) / \cos \sigma_s \\
\bar{\eta}_s &= (\lambda_{\eta} \lambda_s + \mu_{\eta} \mu_s + \nu_{\eta} \nu_s) / \cos \sigma_s
\end{aligned}
\tag{19}$$

#### F. Plate Constants

Let us now assume that we have a photographic plate containing a re-entry trail against the star background with the camera driven by an equatorial



mount so that the stars appear as points on the plate. The trail of the re-entry object will appear as a broken line if a chopping shutter is used.

The first part of the analysis will be the calibration of the plate. This will allow us to find the right ascension and declination of any point on the plate (at least in the region of interest) when the X and Y measurements on the plate are known. A number of stars (probably about 6-15) are identified near the trail; their right ascensions and declinations are found from standard tables for a given date. After corrections are made for proper motion and precession on the tabulated coordinates, we have exact values of  $\alpha_s$  and  $\delta_s$  for each star at the time of the event \*, so that we now know the astronomical coordinates of points on our plate. An approximate position,  $\alpha_c$  and  $\delta_c$  for the plate center is required so that the standard plate coordinates  $\bar{\xi}_s$  and  $\bar{\eta}_s$  can be calculated for each star using equation 19.

Two methods of relating  $\alpha_s$ ,  $\delta_s$  of the star and the X,Y readings on the plate are in common use. The measurements are taken so that the X-axis is parallel to the trail of the re-entry and the Y-axis is perpendicular to the X. In the "four constant solution" we solve the two equations

$$\begin{aligned} a \bar{\xi}_s + b \bar{\eta}_s + c &= X_s \\ b \bar{\xi}_s - a \bar{\eta}_s + d &= Y_s \end{aligned} \quad (20)$$

Two stars at the ends of the trail are used to compute the four constants a,b,c and d and the standard coordinates of any point on the trail may then be found from the inverse transforms:

$$\begin{aligned} \bar{\xi}_s &= A X_s + B Y_s + C \\ \bar{\eta}_s &= B X_s - A Y_s + D \end{aligned} \quad (21)$$

where:

$$\begin{aligned} A &= a / (a^2 + b^2) & B &= b / (a^2 + b^2) \\ C &= - (ac + bd) / (a^2 + b^2) & D &= (ad - bc) / (a^2 + b^2) \end{aligned}$$

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\*Proper motion is corrected to the nearest month and precession to the nearest 1 January.

The "six constant solution" requires a solution of the equations

$$a_x \bar{\xi}_s + b_x \bar{\eta}_s + c_x = X_s; a_y \bar{\xi}_s + b_y \bar{\eta}_s + c_y = Y_s \quad (22)$$

for the constants  $a_x, b_x, c_x, a_y, b_y$ , and  $c_y$  by a least squares method. The standard coordinates can then be found by

$$\bar{\xi} = a_\xi X + b_\xi Y + c_\xi$$

$$\bar{\eta} = a_\eta X + b_\eta Y + c_\eta$$

where:

$$\begin{aligned} a_\xi &= b_y/c_{xy} & a_\eta &= -a_y/c_{xy} \\ b_\xi &= -b_x/c_{xy} & b_\eta &= a_x/c_{xy} \\ c_\xi &= (-b_y c_x + b_x c_y)/c_{xy} & c_\eta &= (a_y c_x - a_x c_y)/c_{xy} \\ c_{xy} &= a_x b_y - a_y b_x \end{aligned} \quad (23)$$

The six constant solution is more complex. However it has the advantage that the X and Y measuring axes need not be exactly perpendicular so that it can be used to correct for any non-orthogonality of a measuring engine. It will also correct for any linear distortions on the plate.

When the plate constants have been found a process of second order corrections is usually applied. These corrections make allowances for errors caused by optical distortions, distortions caused by the emulsion, misplacement of the plate center, errors in the screws, etc. It is now possible to compute theoretical values of X and Y for each star -- i.e., the position where a star of the given right ascension and declination should lie on the plate. From these values residuals are computed from the equations:

$$\Delta X_s = X_s (\text{comp}) - X_s (\text{obs}) \quad \Delta Y_s = Y_s (\text{comp}) - Y_s (\text{obs}) \quad (23')$$

The observed values of X and Y for points on the trail are usually corrected by the equations:

$$X (\text{corr}) = X (\text{meas}) + \Delta X (\text{curve}) \quad Y (\text{corr}) = Y (\text{meas}) + \Delta Y (\text{curve})$$

where  $\Delta X (\text{curve})$  and  $\Delta Y (\text{curve})$  are values found from a smooth curve through the residuals  $\Delta X$  and  $\Delta Y$ . The measured X is the abscissa for the correction

curves and the  $\Delta X$  or  $\Delta Y$  is the ordinate. By examining the residuals it is usually easy to spot an obviously misidentified or misread star since its residuals will be much larger than normal. Often one or more stars can be eliminated and the calibration rerun.

When the plate calibration has been completed, it is possible to relate the X, Y readings of any point on the plate to a position on the celestial sphere. First the standard plate coordinates are found by equation 21 or 23. Direction cosines of the point can then be obtained directly by:

$$\begin{aligned}\lambda &= (\lambda_c + \lambda_{\eta} \bar{\eta} + \lambda_{\xi} \bar{\xi}) / (1 + \bar{\xi}^2 + \bar{\eta}^2)^{1/2} \\ \mu &= (\mu_c + \mu_{\eta} \bar{\eta} + \mu_{\xi} \bar{\xi}) / (1 + \bar{\xi}^2 + \bar{\eta}^2)^{1/2} \\ \nu &= (\nu_c + \nu_{\eta} \bar{\eta}) / (1 + \bar{\xi}^2 + \bar{\eta}^2)^{1/2}\end{aligned}\tag{24}$$

#### G. The Trail Equation

It is assumed in the following reduction that the re-entering vehicle travels in such a path that its projection on the plate is a straight line. If this is not the case the reduction must be done separately on straight line segments of the trail. It is therefore possible to compute a straight line of the form

$$Y = mX + b\tag{25}$$

where m is the slope of the line and b the y intercept; thus only X need be measured directly from the plate after the equation of the line is found. Reading to compute the trail equation is done by dividing the trail into a suitable number of equally spaced (usually between 10 and 20) sections in X and reading the Y corresponding to these readings. Then the X and Y measurements are related by a least squares formula

$$m = (\sum X_i Y_i - n \bar{X} \bar{Y}) / (\sum X_i^2 - n \bar{X}^2) \quad b = \bar{Y} - m \bar{X}\tag{26}$$

where n is the number of points read and  $\bar{Y}$  and  $\bar{X}$  are average values.

#### H. The Pole of the Trail

As discussed above the vehicle appears to travel in a great circle if

the trail is straight along the plate. If the trail is not straight the pole is computed by using points from the early straight line part. The pole of a great circle is, by definition, a point  $90^\circ$  from each point on the great circle. In computing the pole of a trail we will use the beginning and end points (a middle point may be used as a check). It is readily seen that the pole of a great circle through the trail can be found from the cross product of the beginning and end points on the trail. Hence, the direction cosines of the pole  $(\lambda_p, \mu_p, \nu_p)$  are given by

$$\begin{aligned}\lambda_p \sin l &= \mu_b \nu_e - \nu_b \mu_e \\ \mu_p \sin l &= \nu_b \lambda_e - \lambda_b \nu_e \\ \nu_p \sin l &= \lambda_b \mu_e - \mu_b \lambda_e\end{aligned}\tag{27}$$

where  $\lambda_b, \mu_b, \nu_b$  are direction cosines of the beginning point of the trail and  $\lambda_e, \mu_e, \nu_e$  those of the end point, and  $l$  is the length of the trail (in angular measure).

#### I. Relative Coordinates of the Two Stations

It is necessary to use two photographic plates taken from two separate stations to perform a complete optical reduction. These stations will hereafter be called station A and station B. We will now compute the relative positions of the two stations with respect to each other. We will assume that we know their geographic latitudes and longitudes, their elevations above sea level and the exact time of the event. The coordinate system will be that shown in Fig. 7. We must first find the geocentric latitude\*  $\phi'$  of the two stations since we are working in a geocentric system; we will also find the earth's radii  $R_A$  and  $R_B$  for stations A and B, respectively, using the international ellipsoid. We find<sup>5</sup>

$$\phi' = \phi - 11'35''.6635 \sin 2\phi + 1''.1731 \sin 4\phi - 0''.0026 \sin 6\phi$$

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\* Geographic latitude is measured at the earth's surface with reference to the local vertical; geocentric latitude is measured from the center of the earth.



$$\rho = a(0.998320047 + 0.001683494 \cos 2\phi - 0.000003549 \cos 4\phi + 0.000000008 \cos 6\phi)$$

$$a = 6378.388 \text{ Km} = 20926.42796 \text{ Kft}$$

$$R = \rho + \text{elevation of station} \quad (28 \text{ contd})$$

Letting  $\phi_A, \phi_B$  be the geographic latitudes,  $R_A, R_B$  the earth's radii,  $\phi'_A, \phi'_B$  the geocentric latitudes and  $L_A, L_B$  the longitudes of stations A and B respectively, we can find the relative rectangular coordinates in the right ascension-declination system of station B with respect to station A at sidereal time zero at station A. Note: as  $\theta_A = 0$  the x axis goes through the meridian of station A.

$$\begin{aligned} \xi_{AB_0} &= R_B \cos \phi'_B \cos (L_B - L_A) - R_A \cos \phi'_A \\ \eta_{AB_0} &= R_B \cos \phi'_B \sin (L_B - L_A) \\ \zeta_{AB_0} &= R_B \sin \phi'_B - R_A \sin \phi'_A \end{aligned} \quad (29)$$

and obviously

$$R_{AB_0}^2 = \xi_{AB_0}^2 + \eta_{AB_0}^2 + \zeta_{AB_0}^2 = R_{AB}^2$$

Next we must compute the relative coordinates of station B with respect to A at the time of the event  $\theta_A$ , the sidereal time of the event at station A. In general  $\theta_A \neq \theta_B$ . At this time the system will appear as in Fig. 7b. Thus

$$\begin{aligned} \xi_{AB} &= \xi_{AB_0} \cos \theta_A - \eta_{AB_0} \sin \theta_A \\ \eta_{AB} &= \eta_{AB_0} \cos \theta_A + \xi_{AB_0} \sin \theta_A \\ \zeta_{AB} &= \zeta_{AB_0} \end{aligned} \quad (30)$$

We also make use of the direction cosines of the zenithal points for the two stations. From Fig. 7b

$$\begin{aligned} \lambda_{ZA} &= \cos \phi_A \cos \theta_A & \lambda_{ZB} &= \cos \phi_B \cos \theta_B \\ \mu_{ZA} &= \cos \phi_A \sin \theta_A & \mu_{ZB} &= \cos \phi_B \sin \theta_B \\ \nu_{ZA} &= \sin \phi_A & \nu_{ZB} &= \sin \phi_B \end{aligned} \quad (31)$$

where  $\phi$  is now the geographic latitude.

### J. The Radiant

The radiant of a meteor or other moving body is defined as the direction from which it appears to have originated. In our case this point is easily seen to be the point at which the projected great circles defined by the trails on the two plates intersect. The angle between the two great circles is the same as the angle between their poles; the radiant is the cross product of their poles. Hence, using equation 27 for stations A and B

$$\begin{aligned}\lambda_R \sin Q &= \mu_{AP} \nu_{BP} - \nu_{AP} \mu_{BP} \\ \mu_R \sin Q &= \nu_{AP} \lambda_{BP} - \lambda_{AP} \nu_{BP} \\ \nu_R \sin Q &= \lambda_{AP} \mu_{BP} - \mu_{AP} \lambda_{BP}\end{aligned}\tag{32}$$

where  $Q$  is the angle of intersection between the two meteor trails as seen from station A and station B. The radiant chosen must lie above the horizon; this is done by requiring the dot product  $\bar{Z} \cdot \bar{R}$  to be positive or zero, ( $\bar{Z}$  being a vector in the zenith direction).

### K. The Computation of Range

Since the trajectory of the re-entering vehicle is assumed to lie on a great circle only one possible plane exists through the re-entry track and the observing station; the pole for that great circle is perpendicular to this plane. The distance  $S_A$  from the first station (A) to the plane determined by the missile and the second station (B) is seen to be (Fig. 8) the projection of  $R_{AB}$  (equation 29) onto a perpendicular to the plane, i.e., onto the line through the pole. Hence, using the direction cosines of the pole (equation 27) we get the distance

$$S_A = \xi_{AB} \lambda_{BP} + \eta_{AB} \mu_{BP} + \zeta_{AB} \nu_{BP}\tag{33}$$

The distance or range  $R_{Ai}$  of station A to any point on the plane is given by  $R_{Ai} = S_A \sec \epsilon_i$  where  $\epsilon_i$  is the angle between  $\bar{P}_B$  and the range vector  $\bar{R}_{Ai}$ . Since points on the re-entering track are just specific points on this plane we have the equation

$$\cos \epsilon_1 = \lambda_{Ai} \lambda_{BP} + \mu_{Ai} \mu_{BP} + \nu_{Ai} \nu_{BP}$$

so

$$R_{Ai} = S_A / \cos \epsilon_1 \quad (34)$$

#### L. The Computation of Height

In computing height above sea level, the height  $h_{Ai}$  above a plane through the station normal to the zenith direction is first computed. The zenith distance  $Z_{Ai}$  for any point on the re-entry is given by

$$\cos Z_{Ai} = \lambda_{ZA} \lambda_{Ai} + \mu_{ZA} \mu_{Ai} + \nu_{ZA} \nu_{Ai} \quad (35)$$

see Fig. 9. Thus the height above the plane normal to the zenith is found by

$$h_{Ai} = R_{Ai} \cos Z_{Ai} \quad (36)$$

To obtain height  $H_i$  above sea level, the elevation of the station  $H_A$ , and a correction  $\delta h_{Ai}$  for the earth's curvature must be added. As in Fig. 9 the total height of the vehicle from the center of the earth is given by

$$(\rho_A + h_{Ai} + \delta h_{Ai})^2 = \rho_A^2 + R_{Ai}^2 + 2\rho_A R_{Ai} \cos(Z_{Ai})$$

and thus solving for  $\delta h_{Ai}$

$$\delta h_{Ai} = (\rho_A^2 + R_{Ai}^2 + 2\rho_A h_{Ai})^{1/2} - (\rho_A + h_{Ai}) \quad (37)$$

The total height above mean sea level is

$$H_i = h_{Ai} + H_A + \delta h_{Ai} \quad (38)$$

where  $H_A$  is the elevation of the station.

#### M. The Calculation of Distance Along the Trail

After the ranges have been computed for any two points on the re-entry track the distance between them can easily be found. The components of the vector  $R_{Ai}$  in the astronomical equatorial system are

$$\begin{aligned} \xi_{Ai} &= R_{Ai} \lambda_{Ai} \\ \eta_{Ai} &= R_{Ai} \mu_{Ai} \\ \zeta_{Ai} &= R_{Ai} \nu_{Ai} \end{aligned} \quad (39)$$

Letting the first point seen on the trail be called  $D_{A1}$  we can find the distance from the beginning of the trail to any other point by

$$D_{Ai} = \left[ (\xi_{Ai} - \xi_{A1})^2 + (\eta_{Ai} - \eta_{A1})^2 + (\zeta_{Ai} - \zeta_{A1})^2 \right]^{1/2} \quad (40)$$

#### N. The Computation of Relative Time

Except in special cases where a common point can be identified along the trail in both photographs, it is impossible to find exactly simultaneous points on the two plates; also, no absolute time is available for any point. However, if one or more of the photographs has been chopped, a relative time (i.e., time measured from an arbitrary zero point) can be computed and makes possible the calculations of height and range as functions of time and also an estimate of the velocity of the vehicle as the time derivative of distance along the trail.

The cameras used to compute relative time employ a shutter which opens and closes at known constant intervals, thus making chops on the plate. The coordinates of the mid-point of each chop are read; these points correspond to known relative times since the chopping rate is known.

If  $P$  is the period of revolution of the rotating shutter and  $N$  the number of occultations, all equally spaced, per revolution, the instant  $t_i$  corresponding to the  $i$ 'th break is

$$t_i = i \left( \frac{P}{N} \right) + \Delta t_i \quad (41)$$

where  $\Delta t_i$  is a function of the position of the re-entry body on the plate relative to the center of rotation of the shutter. For calculations not involving Super-Schmidt cameras it will be assumed that  $\Delta t_i$  is negligible.

For Super-Schmidt cameras the correction is often fairly large. As a first approximation let us assume that the projection center  $(X_c, Y_c)$  very nearly coincides with the center of rotation of the shutter  $(X_q, Y_q)$ ; in practice this approximation is usually valid for a distance between the two centers less than one cm. In Fig. 10 let  $X_i, Y_i$  be the coordinates of the  $i$ 'th point on the trail. The angle that the blade edge makes with the  $i$ 'th dash is obviously given by



$$\tan \omega_i = (X_i - X_q) / (Y_i - Y_q) \quad (42)$$

If  $\omega_0$  is the value of  $\omega$  at the origin of the time scale we have

$$\Delta t_i = \pm (\omega_i - \omega_0) P/2\pi \quad (43)$$

where the + sign is used when the shutter edge chases the re-entry body as in Fig. 10, since here more than one revolution of the shutter occurs between chops and the - sign is used when the edge runs toward the body.

If the distance between the center of rotation of the shutter and the projection center is too large for equation 42 to be valid an exact expression for  $\tan \omega_i$  has been found by Whipple and Jacchia<sup>1</sup> to be

$$\tan \omega_i = z \left[ u (X_i - X_q) - (Y_i - Y_q) \right] / \left[ u (Y_i - Y_q) + (X_i - X_q) \right] \quad (44)$$

where

$$u = (Y_q - Y_c) / (X_q - X_c)$$

$$z^2 = 1 + (1/f^2) \left[ (X_q - X_c)^2 + (Y_q - Y_c)^2 \right]$$

and  $f$  is the focal length of the camera.

#### 0. The Problem of Stationary Cameras

One or both of the photographs used in the analysis may have been made by a stationary camera. In this case the star image consists of a trail rather than a point on the plate. The ends of this trail correspond to the opening and closing of the camera. Thus the end of the star trail where the measurement is made is not simultaneous with the time of the event. The declinations of the stars are unchanged but the right ascensions are shifted by an amount  $\Delta\theta = \theta_A - \theta'_A$  where  $\theta_A$  is the sidereal time of the event and  $\theta'_A$  the time at the end of the star trail where the plate is measured. Therefore the direction cosines ( $\lambda'_{Ai}, \mu'_{Ai}, \nu'_{Ai}$ ) computed by equation 4 must be changed to the true direction cosines by

$$\begin{aligned} \lambda_{Ai} &= \lambda'_{Ai} \cos \Delta\theta - \mu'_{Ai} \sin \Delta\theta \\ \mu_{Ai} &= \mu'_{Ai} \cos \Delta\theta + \lambda'_{Ai} \sin \Delta\theta \\ \nu_{Ai} &= \nu'_{Ai} \end{aligned} \quad (45)$$

It is easily shown that if only the direction cosines of the plate center and of the  $\xi$  and  $\eta$  axes are corrected by equation 45 from the time of the star trail's end to the time of the event, the direction cosines of each point on the re-entry trail computed from equation 24 will be correct.

### III. THE EXPERIMENTAL OPTICAL PROGRAM

The optical reduction method outlined in the present paper for determining the space trajectory of a re-entering body requires that the event be photographed simultaneously from two stations separated by a relatively long baseline. If velocity is to be computed, at least one of the cameras must be equipped with a chopping shutter so that relative time may be obtained. The Trailblazer program utilizes two Baker Super-Schmidt cameras, one at Arbuckle Neck, the other at Eastville. In addition, photographs taken by NASA on ballistic cameras from their Wallops Island and Coquina Beach stations are usually available. Figure 11 shows the relation of these camera sites to each other and to the launch site.

#### A. The Super-Schmidt Camera

Figure 12a is a photograph of a Baker Super-Schmidt camera used to obtain the re-entry photographs. This camera has an aperture of 12.25 inches, a focal length of 8 inches, an effective focal ratio of 0.85 and a field of view of approximately 58 degrees. It is supplied with an interchangeable shutter and motor capability, so that, depending upon the number of blades in the shutter and the rpm of the motor, different chopping speeds are possible. The surface of the shutter is immediately in front of the photographic emulsion.

A schematic diagram of the optical system appears in Fig. 12b. Two part-hemispherical correcting shells, concentric with the center of curvature of the spherical mirror, provide almost complete correction of spherical aberration. Any remaining aberrations and achromatism over the range 3800-7000 Å are corrected for by an aspherical correcting plate through the center of curvature. The spherical focal surface, concentric with the rest of the

system, lies just within the second shell. A light ray entering the system passes first through the outer shell, the correcting plate, and the second shell before it is reflected at the mirror; it then passes through the inner shell a second time before it reaches the focal surface.

Since the focal surface of the Super-Schmidt is spherical, flat film must be molded to this shape. The film is held in the focal plane, convex toward the mirror, by a partial vacuum (about 20" Hg) between the film and the focal plane. A special, very fast, blue sensitive X-ray film is usually employed in these tests. It is made by Eastman Kodak especially for the Harvard Meteor Project.

The optical projection in the Super-Schmidt camera is very nearly a truly concentric spherical projection; this allows the star images to maintain their relative positions on the celestial sphere without distortion. The curved film is projected onto a flat plate by a very nearly gnomonic projection; this flat plate (a positive) is then used for the analysis.

Although much of our work is done with the Super-Schmidt cameras the reduction methods and the computation program discussed here will work with any set of ballistic plates.

#### B. Star Identification

At each station the re-entry trail and the field stars are photographed simultaneously. If the photograph is taken by a camera equipped with an equatorial mount, such as the Super-Schmidt, the stars appear as points on the plate (Fig. 13a); if the camera is stationary, the stars will appear as streaks (Fig. 13b). However, if the time of opening and closing of the shutter is known the ends of these streaks are identifiable in time and may be used in the analysis as discussed in Section II-0.

The approximate coordinates of the plate center in right ascension and declination are recorded by the camera operator; the direction of north on the photograph is also indicated. Thus the general position and orientation of the photograph are known. For Super-Schmidt plates, in particular, the use of Norton's Star Atlas for a preliminary identification of the brighter stars is recommended since the scale of its star maps is very nearly that of

the Super-Schmidt film. By overlaying the film on the charts, many of the stars may be immediately identified and the same stars easily found on the BD or CD chart, thus outlining the area of interest. The position of the trail among the stars is then found on the BD or CD charts and may be sketched in lightly for future reference.

Stars to be measured should be chosen at uniform intervals along the trail, (X-direction) and alternately on opposite sides of the trail (+ and - Y). In order to minimize centering errors the stars chosen should be of nearly the same photographic magnitudes and as near the plate limit as possible. The stars to be measured may be marked on the star charts and their BD or CD numbers as well as their approximate coordinates recorded from the BD or CD tables. When the BD or CD number of a star is known its position may be found in the more exact star catalogues.\* The following information should be recorded for each star:

Right Ascension

Declination

Equinox of Reference

Epoch of Observation

Components of Proper Motion in Right Ascension and Declination.

The stars to be measured may be marked on the glass side of the plate. About 8 to 10 stars are usually sufficient for a plate calibration; at least 6 should always be used.

### C. The Position of the Plate Center

In addition to the coordinates of the selected stars, the coordinates of the plate center must be estimated. Four stars are chosen such that two lines intersecting at the center may be drawn through the stars (i.e., each two stars will lie on a line through the center; the stars should be on opposite sides of the center). The four stars are identified on the star charts and the lines drawn lightly on the charts. The point of intersection of the two lines on the star chart may be read off as the approximate right

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\* A list of star catalogues will be found in Appendix A.



ascension and declination of the plate center.

#### D. The Measurement of the Plates

The plates should be measured on a measuring engine equipped with two mutually perpendicular precision screws so that the X and Y coordinates may be measured simultaneously in a plane (Fig. 14). The plate should be oriented in the measuring engine so that the trail of the re-entry is parallel to the X axis; if the trail is not completely straight, the X axis should be made parallel to the straight line portion. All measurements of the plate must be made without removing the plate from the machine. To avoid possible backlash effects, all measurements are made with the screw turning in the same direction (customarily toward increasing scale readings). To check the consistency of the readings over a period of time, two points are selected near the ends of the trail and read at occasional intervals during the measuring of the plates. In this way any disturbances of the machine by temperature changes or mechanical vibration will show up. Readings are made to the nearest micron.

Various methods of finding the coordinate of a point are possible -- usually an average of three settings is sufficient, as long as the readings do not vary by more than a few microns. It is sometimes advisable to repeat a series of measurements with the plane rotated  $180^\circ$  to check for possible systematic or accidental errors. Whatever method of averaging is used, only one value of each point is used in the program. Therefore any averaging or statistical analysis on the readings must be performed prior to entry into the program.

When the plate has been adjusted in the machine so that the X-axis is parallel to the re-entry trail, the center point of each star selected for analysis is read to get the coordinates in X and Y of the star. Next, the trail, starting from the first distinct point and ending at the last distinct point (whether or not these points are seen as dashes) is divided up into from about 10 to 20 equal parts (in X). The Y measure at each of these equally spaced X's is read for computing the trail equation.

If the re-entry event is photographed with a camera equipped with a chopping shutter the re-entry trail will appear as a fairly straight dashed line -- the dashes being caused by the shutter breaks (Figs. 13a and 13b). If the camera does not have a chopping shutter, the re-entry trail will appear as a streak among the stars (Fig. 13c). The X-measurement of the middle of each dash should be taken in the case of a chopped plate. The Y coordinate may then be computed for each dash by the trail equation so that Y need not be read for the dashes. On a streak photograph any point of special interest, such as a flare, should be measured. Also, the very first and very last points visible on the trail must be measured. At least five measurements should always be made along the trail.

A diagrammatic sketch of the required measurements is shown in Fig. 15. This diagram is only theoretical and does not represent a particular re-entry. Stars appear over the entire plate, but only those near the trail are of interest here. Stars labeled A-H have been selected for measurement. These stars are seen to be selected on alternate sides of the trail and relatively evenly spaced along the trail. Star F is seen to lie on the trail itself. The actual trail lies between the two lines marked T.B. and T.E. (beginning and end); it will be noticed that many dashes are missing at the start of the trail and a few near the end. The measurements for the trail equation will be made at equal X increments between TDB and TDE, the first and last distinct points. In this case the interval is divided into 10 equal increments (in X) labeled T1, T2...T11. Notice that the reading is taken along the trail and that the point read may lie between two dashes. A Y measure is taken for each of the 11 X's. The first point T.B. and the last point T.E. are also measured.

The center of each dash should next be read in X starting with the first distinct dash, D1 and continuing for the rest of the distinct dashes (one or more may be missing as is dash 18 in the diagram). The flare occurring at dash 17 is particularly interesting since it may also show on the other plate and so could be used as a common point. The plate center should also be measured in X and Y.



## E. Computations

After the measurements have been made, the next step is a plate calibration which will relate the X, Y measurements on the plate to positions in the sky. The plate calibration program is run using as input the measured X, Y of the known stars and the coordinates of the stars from the catalogues. The output consists of inverse plate constants, and direction cosines for the plate center and the coordinate axes which are needed as input to the optical trajectory program. When the results of the plate calibration program are available, the residuals (or  $\Delta X$  and  $\Delta Y$ ) for each star are examined; if any of them are extremely large, the calibration is repeated omitting the offending star. When the calibration is satisfactory, i.e., the residuals are very small, the residuals may be plotted against X (Fig. 16). If the residuals are within the limits of measurement they are usually ignored; in other cases they are applied to each measurement along the trail as discussed in Section II-F.

When both plates have been calibrated by the plate calibration program, the input cards to the optical trajectory program are made as discussed in Section VI. The optical trajectory program uses the results of the calibration program, the geographic coordinates of the stations, the time of the event, data about the cameras, the measured X, Y readings taken at equal X increments for computing the trail equation and the X readings for each measured point along the trail from the two plates and has as output such meaningful parameters as range, height, direction cosines, and distance along the trail for each measured point along the trail.

## F. The Reduction of Trailblazer Ik

To demonstrate the use of the optical trajectory program the reduction of Trailblazer Ik will be discussed. Trailblazer Ik was fired from Wallops Island on 27 July 1962 at 21:50:00 E.S.T. and was photographed by the Super-Schmidt cameras at Arbuckle Neck and at Eastville at 21:56:21.583 (Figs. 17a and 17b). The plates were read at Harvard University on a Mann Measuring Engine as discussed above.

The astronomic coordinates and the X, Y measurements of the stars are given in Table II for the SL (Arbuckle Neck) camera. The coordinates have already been corrected for proper motion. Care must be taken in using the values of proper motion given in the star catalogues, since some catalogues give proper motion as  $\Delta \alpha$  and some as  $\Delta \alpha \cos \delta$  per year or per century. Corrections in proper motion are made to the nearest month. Also, the X, Y measures have been averaged to give one reading for each point.

TABLE II

Stars Identified and Measured for the SL Photograph  
of Trailblazer IK taken at Arbuckle Neck

Name	Right Ascension		Declination		Equinox	X mm	Y mm
A	20h 6m	13.754s	-28° 35'	13.95"	1950	173.1015	20.411
B	20h 7m	28.968s	-29° 54'	35.97"	1950	178.155	20.738
C	20h 8m	17.23 s	-30° 56'	52.6 "	1950	182.086	21.1255
D	20h 13m	45.23 s	-32° 45'	45.2 "	1950	189.9875	18.8385
E	20h 14m	25.78 s	-33° 50'	22.2 "	1950	194.171	19.4495
F	20h 17m	2.57 s	-34° 44'	34.3 "	1950	198.177	18.4505
Plate Center	19h 45m		-18° 17'		1855	133.205	23.520

The first step is to perform a calibration of the plate. (We will discuss only the reduction of the SL data as the SS data are reduced in an identical manner.) From Table II we see that we have a total of 6 stars all of which have their coordinates referred to the equinox of 1950. We also see that the coordinates of the plate center are referred to the equinox of 1855. Thus two sets of precession constants must be used; the first will precess the coordinates of the plate center from 1855-1963 and the second will precess the coordinates of the stars from 1950-1963 (the nearest 1 January). The correct precession constants are shown in Table V.  $\Delta \theta$  will be zero since

the Super-Schmidt cameras track the stars. We will run one four constant solution, using stars A and F (or 1 and 6). The card set-up and a sample computer print-out are given in Appendix B.

Upon inspection of the results of the calibration program, it is seen that the residuals are very small and within the accuracy of measurement. Therefore, no corrections need be applied to the measured values of X and Y. The six constant solution will be used in the reduction. The next step is to prepare the cards for the Optical Trajectory Program.

The event took place at 21:56:21.583 E.S.T. on 27 July 1962. Converting to Greenwich time, we have 02:56:21.583 G.M.T. on 28 July 1962. We must therefore find the  $\theta_0$  for 28 July in the 1962 Almanac since the date at Greenwich must be used. It is found to be 20h 20m 43.038s. The Greenwich time and geographic coordinates are given on the next card.

From Table III we see that 15 equally spaced points were read along the trail. The first X point is at 176.000mm and an average Y on the trail is about 20.140mm. These are subtracted from each reading along the trail and the differences are inserted into the program. This set of cards will be used to compute the trail equation to give Y as a function of X.

The next two cards will give the inverse plate constants (from Page 4 of our print-out for the six constant solution) and the next three will give direction cosines of the plate center and standard axes (from Page 5 of the print-out). The names of the variables in the plate calibration print-out are identical to those used in the card write-up (Section V).

The next card gives the X coordinates of 5 points of special interest (the first point must be the physically first point read on the trail, T.B. in our diagram and the last point must be the physically last point, T.E.). The other three points may be any intermediate points such as flare points or other points of special interest.

The next cards will give information about the dashes as shown in Table IV. Table IV is not complete but enough dashes are shown to give an understanding of the card set up.

TABLE III

X, Y Readings at Equal X Increments Used in Computing  
the Trail Equation Data from the SL Photograph  
of Trailblazer Ik Taken at Arbuckle Neck

X	Y	$\Delta X$	$\Delta Y$
mm	mm	mm	mm
176.0	20.140	0	0.000
177.5	20.141	1.5	0.001
179.0	20.148	3.0	0.008
180.5	20.142	4.5	0.002
182.0	20.146	6.0	0.006
183.5	20.142	7.5	0.002
185.0	20.139	9.0	-0.001
186.5	20.140	10.5	0.000
188.0	20.144	12.0	0.004
189.5	20.142	13.5	0.002
191.0	20.140	15.0	0.000
192.5	20.139	16.5	-0.001
194.0	20.137	18.0	-0.003
195.5	20.140	19.5	0.000
197.0	20.136	21.0	-0.004

End Point      X = 175.861  
                    Y = 198.644

TABLE IV

X Readings for Dashes for the SL Photograph  
of Trailblazer Ik taken at Arbuckle Neck

Dash #	Wt.	X
		mm
1	4	175.946
2	4	176.245
3	1	176.534
4	2	176.796
5	1	177.057
6	2	177.332
87	1	198.567

TABLE V

## Precession Constants

	1855-1963	1950-1963
$X_x$	0.99965376	0.99999498
$X_y$	-0.02412862	-0.00290553
$X_z$	-0.01049490	-0.00126316
$Y_x$	0.02412862	0.00290553
$Y_y$	0.99970885	0.99999578
$Y_z$	-0.00012664	-0.00000183
$Z_x$	0.01049490	0.00126316
$Z_y$	-0.00012661	-0.00000183
$Z_z$	0.99994490	0.99999920



Table VI gives information about the cameras and is used in making out the remaining cards. A similar set of cards must be made for the SS camera. Appendix C gives a listing of cards used as input to the trajectory program, and a sample computer output. Not all of the card set up and print-out are shown, since Appendix C is meant to show a sample calculation and not give useful values of a particular shot.

TABLE VI

Camera Data for Trailblazer Ik

	SL Camera	SS Camera
XQ	133.205	126.656
YQ	23.520	10.333
XC	133.205	126.656
YC	23.500	10.333
$F^2$	4074	4074
Period	0.1	0.1
Occult	2	2
Direction	Clockwise	Counter Clockwise

#### IV. THE PLATE CALIBRATION PROGRAM

The plate calibration program is designed to carry out the computations necessary to relate X, Y measurements on the photographic plate to astronomical coordinates and to compute residuals for the correction of the measurements of points along the trail. The plate constants are computed by both the four and the six constant solutions and the method giving the best (i.e., smallest or most consistent) residuals is used in the later work.

The input to the program includes the right ascension and declination and the X, Y measurements of each star selected and of the plate center. The astronomical coordinates of the stars are corrected for proper motion to the nearest month before being inserted into the program; however, the corrections for precession to the nearest 1 January are performed within the program in direction cosine form. A generalized flow chart of the plate calibration program and of subroutine SIXCON appears in Fig. 18.

The computer output consists of the following print-outs:

- Page 1 Listing of the input data for the plate center and each star in easily understandable form
- Page 2 Direction cosines for the uncorrected ( $\Delta\theta = 0$ ) plate center and the coordinate axes  
Direction cosines and standard coordinates ( $\sigma, \bar{\eta}, \bar{\xi}$ ) for each star
- Page 3\* Plate constants, inverse plate constants, and residuals for the four constant solution (each four constant solution will be on a separate page)
- Page 4\* Plate constants and inverse plate constants and residuals for the six constant solution
- Page 5\* Corrected direction cosines for the plate center and coordinate axes (Note: since in our case  $\Delta\theta = 0$ , the direction cosines on Page 5 are identical with those on Page 2).

Appendix B gives a print-out sample of the results of the plate calibration program.

The residuals should be inspected to see if any star gives either very large (probably above 100 microns) or very inconsistent values. Such a star should be eliminated and the calibration program rerun. When the residuals seem to be small and lie on a smooth curve (to within the accuracy of measurement), they should be plotted against the X reading.

---

\* If more than one four constant solution is run, they will appear on pages 3', 3'', etc.

Fig. 16 is a sample curve of the residuals for a theoretical case. A smooth curve is drawn through the residuals and this curve (one for  $\Delta X$  and another for  $\Delta Y$ ) is used to correct the sets of X, Y readings to be used in computing the trail equation and the set of X readings measured at points to be reduced (equation 23').

A listing of the plate calibration program appears in Appendix D. The six constant solution is performed by subroutine SIXCON which solves the matrix equation

$$A X = B$$

where

$$\begin{aligned} A &= \begin{pmatrix} \sum \bar{\xi}_i^2 & \sum \bar{\xi}_i \bar{\eta}_i & \sum \bar{\xi}_i \\ \sum \bar{\xi}_i \bar{\eta}_i & \sum \bar{\eta}_i^2 & \sum \bar{\eta}_i \\ \sum \bar{\xi}_i & \sum \bar{\eta}_i & N \end{pmatrix} \\ X &= \begin{pmatrix} a_x & a_y \\ b_x & b_y \\ c_x & c_y \end{pmatrix} \\ B &= \begin{pmatrix} \sum x_i \bar{\xi}_i & \sum y_i \bar{\xi}_i \\ \sum x_i \bar{\eta}_i & \sum y_i \bar{\eta}_i \\ \sum x_i & \sum y_i \end{pmatrix} \end{aligned} \quad (46)$$

using library routine XSIMEQF. This routine solves equation 46 for X by triangularization of the A matrix. If the solution is successful, the X matrix will replace the original A matrix in storage. In the program the word MATRIX is a code which equals 1 for a successful solution of the equation, equals 2 for overflow or underflow, and equals 3 for a singular matrix. The arguments used in calling the subroutine XSIMEQF together with their values for this example are:

Arg<sub>1</sub> = maximum value of subscript I of A (I, J) = 10  
 Arg<sub>2</sub> = number of rows in matrix A = 3  
 Arg<sub>3</sub> = number of columns in matrix B = 2  
 Arg<sub>4</sub> = name of matrix A = A  
 Arg<sub>5</sub> = name of matrix B = B  
 Arg<sub>6</sub> = scale factor for use in calculation = 1  
 Arg<sub>7</sub> = one dimensional erasable array for use in solution = TWO = 10

A more detailed description of the subroutine XSIMEQF is available.

The data cards are set up as follows:

#### Data Cards for the Plate Calibration Program

Card 1                      Column 1 contains a "1" and columns 2-72 contain a Hollerith title to be written on tape A3. This title may contain any identifying information desired.

Card 2                      Contains two fixed point numbers (format 2I5)  
                             NN = number of sets of precession constants used in the reduction  
    in columns 1-5 (format I5) Up to 5 sets may be used  
                             N = number of stars used in the reduction in columns 6-10  
    (format I5).

The next group of cards will consist of 4 cards for each of the NN sets of precession constants.

The first card of each precession set will be a Hollerith date card constructed as follows:

A zero in column 1  
 Year of equinox of table in columns 2-5  
 Hollerith hyphen in column 6  
 Year of event to 1 January in columns 7-10.

The next 3 cards of the precession set will each have 3 precession constants in floating point form (format 3E15.8) as follows:

Precession card 2  
 X<sub>x</sub> in columns 1-15  
 X<sub>y</sub> in columns 16-30  
 X<sub>z</sub> in columns 31-45

Precession card 3

$Y_x$  in columns 1-15

$Y_y$  in columns 16-30

$Y_z$  in columns 31-45

Precession card 4

$Z_x$  in columns 1-15

$Z_y$  in columns 16-30

$Z_z$  in columns 31-45

Each precession set will also have a number J associated with it; J being assigned according to the placement of the set in the deck, i.e., J = 1 for the first precession set read in, J = 2 for the second precession set, etc. The precession set to be used in each case will be that one corresponding to the equinox for which the coordinates of the star (or plate center) are recorded in the standard tables.

The next card will have 10 numbers (formats 2F4.0, F8.4, 2F4.0, F7.3, 2F10.5, I5, E15.8) which give the coordinates of the plate center as follows:

Right ascension in hours	in columns 1-4
Right ascension in minutes	in columns 5-8
Right ascension in seconds	in columns 9-16
Declination in degrees	in columns 17-20
Declination in minutes	in columns 21-24
Declination in seconds	in columns 25-31
X reading of plate center in mm	in columns 32-41
Y reading of plate center in mm	in columns 42-51
J = number of precession set to use	in columns 52-56
$\Delta\theta$ = correction of plate center for stationary cameras	in columns 57-71

(Note:  $\Delta\theta = 0$  for tracking cameras.)

The next N cards will give the coordinates of the N stars to be used in the analysis and will have 9 numbers per card (formats 2F4.0, F8.4, 2F4.0, F7.3, 2F10.5, I5). Corrections in right ascension and declination



for proper motion must be made before the values are punched on the cards. The N cards will have the following format:

Right ascension in hours	columns 1-4
Right ascension in minutes	columns 5-8
Right ascension in seconds	columns 9-16
Declination in degrees	columns 17-20
Declination in minutes	columns 21-24
Declination in seconds	columns 25-31
X reading of plate in mm	columns 32-41
Y reading of plate in mm	columns 42-51
J = number of precession set to use	columns 52-56

Each star will have a number L associated with it depending on its placement in the deck, i.e., L = 1 for the first star card read in, L = 2 for the second card, to L = N for the last star card read in.

The next card will have one number MM (format I5) in columns 1-5; MM is the number of four constant solutions to be run. (Note: it may be desired to try different combinations of stars as end points in the four constant solution; this card allows for this possibility.)

The last MM cards will each have 2 numbers (format 2I5), the first number is the L for the star used as the beginning point in the four constant solution in columns 1-5 and the second number is the L for the star to be used as end point in columns 6-10.

## V. THE OPTICAL TRAJECTORY PROGRAM

The optical trajectory program computes such physically meaningful parameters as range, height, direction cosines, distance along the trail, and relative time for the X, Y measurements of two photographic plates of the re-entry event. Cards may be made for a future velocity calculation; however, velocity is not computed by the present program.

Input to the program includes the sets of X, Y readings taken at equal X increments along the trail to be used to compute the trail equation, the sets of X readings taken at selected intervals along the trail (such



as the centers of the dashes) where a reduction is desired, the time of the event, the geographic coordinates of the camera stations, the plate calibration data obtained from running the calibration program for the two plates, and information about the cameras used to photograph the event. Only one position measurement is entered for each point read; any averaging or residual corrections necessary to obtain this measurement is done prior to entry into the program.

A generalized flow chart of the optical trajectory program and of each subroutine appears in Fig. 19. The program itself is given in Appendix E.

The computer output consists of the following print outs:

1. A listing of the input data
2. Time (Greenwich and sidereal) and geographic coordinates of the two stations; direction cosines for vector from station 1 to station 2 at sidereal time zero and at the time of the event
3. Plate calibration data, trail equation, X, Y, and direction cosines of the five points and direction cosines of the pole for each station
4. Direction cosines of the radiant
5. Ranges and heights for the five points
6. Tables of X, Y, range, cos Z, and height above mean sea level for each dash for the two stations
7. Tables of relative time, range, height, and direction cosines in the altitude-azimuth system for each dash for the two stations

Note: Certain checks have been performed in the program and the small errors found printed out for inspection. For example, checks have been made to determine if the sum of the squares of the direction cosines of a point equals one.

A sample output is shown in Appendix C for the Trailblazer Ik re-entry.

A card output containing values of distance along the trail, relative time, range, height, direction cosines in the altitude-azimuth system, dash number and dash weight is also available.

The dash weight gives a measure of the confidence level that the person reading the plate put on that dash. The weights are assigned as follows:

1	very poor
2	poor
4	fair
6	good
8	excellent

The weights are not used in the calculations of this program but are included in the card output for possible use in future velocity calculations.

The following is a description of the input cards necessary for running the optical trajectory program.

Card 1 Column 1 contains a "1" and columns 2-72 contain a Hollerith title to be written on tape A3. This title may be any identifying information desired.

Card 2 will contain three numbers (formats 2F4.0, F8.4 )

Theta0	in hours	in columns 1-4
Theta0	in minutes	in columns 5-8
Theta0	in seconds	in columns 9-16

Theta0 is the sidereal time at Greenwich for 0h Universal Time for the day of the event at Greenwich (the day will be defined to start at 0000 Universal Time); the theta0 is found in the table of "Universal and Sidereal Times" of The American Ephemeris and Nautical Almanac for the proper year - theta0 is tabulated separately for each day of the year.

Following cards 1 and 2 which give general information about the event, we will have two sets of cards, one for each station. In the following discussion the making of these cards for station A will be shown; those for station B are made in exactly the same way and immediately follow those of station A in the deck.

Card 3 will have a zero in column 1 and any Hollerith identification of station A desired in columns 2-72.

Card 4 will have 10 numbers giving the coordinates of station A (format 3(2F4.0, F8.4), F11.7)

Event time (Greenwich)	in hours	in columns 1-4
Event time	in minutes	in columns 5-8
Event time	in seconds	in columns 9-16
Latitude of Station A	in degrees	in columns 17-20
Latitude	in minutes	in columns 21-24
Latitude	in seconds	in columns 25-32
Longitude of Station A	in hours	in columns 33-36
Longitude	in minutes	in columns 37-40
Longitude	in seconds	in columns 41-48
Elevation of Station A	in kilofeet	in columns 49-59

The next group of cards will be used to compute the trail equation by the least squares method. A starting  $X_0$  and  $Y_0$  are chosen and the calculation performed on the differences from these values where

$$\Delta X = X_i - X_0$$

$$\Delta Y = Y_i - Y_0.$$

The  $X_0$  chosen is the first X reading on the trail, the  $Y_0$  is about the average of the Y readings along the trail.

Card 5 contains a single number N which is the number of points along the trail (format I5) in columns 1-5.

Card 6 contains two numbers (format 2 E15.8)

$X_0$  in columns 1-15

$Y_0$  in columns 16-30.

Card 7 etc. The next  $N/2$  or  $(N/2) + 1$  cards contain the values of

$\Delta X_i$  and  $\Delta Y_i$  punched two to a card (format 4 E15.8)

$\Delta X_i$  in columns 1-15

$\Delta Y_i$  in columns 16-30

$\Delta X_{i+1}$  in columns 31-45

$\Delta Y_{i+1}$  in columns 46-60

The next five cards give calibration data obtained as output from the plate calibration program; each card (except the last which has only two numbers) has three numbers (format 3 E15.8). These values are given

as output from the plate calibration program where they are labeled by the same symbol as in this description.

Card 1 of the 5

AEXI	in columns 1-15
BEXI	in columns 16-30
CEXI	in columns 31-45

Card 2 of the 5

AETA	in columns 1-15
BETA	in columns 16-30
CETA	in columns 31-45

Card 3 of the 5

LAMC	in columns 1-15
LAMEXI	in columns 16-30
LAMETA	in columns 31-45

Card 4 of the 5

MUC	in columns 1-15
MUEXI	in columns 16-30
MUETA	in columns 31-45

Card 5 of the 5

NUC	in columns 1-15
NUETA	in columns 16-30

The next card gives 5 values of X; the first X on this card must be the X reading at the physically first point of the trail and the fifth X on this card must be X reading at the physically last point of the trail. The other three X's should be common points if any exist or any other particularly interesting value of X. Note: this card must always contain 5 X's whether or not any common points are found; also, the first and last X on the card must be the physically first and last X read on the event (format 5F10.5).

The next card will contain one number (format I5) which will be the number of dashes read along the trail plus 2 (for the end points of the

trail which in general are not the middle of dashes)

NUM = number of dashes + 2      in columns 1-5

The next group of cards will give the information for each dash with three readings to a card. The first reading will correspond to the beginning of the trail and the last to the physically last point on the trail (these two readings will not, in general, be read from the middle of the dashes). These two end points will have a dash number of 0 and a weight of 1 (format 3 (2I5 , F10.5)).

Dash number	in columns 1-5
Weight of dash	in columns 6-10
X reading of dash	in columns 11-20
Dash number	in columns 21-25
Weight of dash	in columns 26-30
X reading of dash	in columns 31-40
Dash number	in columns 41-45
Weight of dash	in columns 46-50
X reading of dash	in columns 51-60

The next and last group of the set gives information about the cameras used.

Card 1 of the group contains four numbers (format 4 E15.8)

Observed X coordinate of shutter center XQ	in columns 1-15
Observed Y coordinate of shutter center YQ	in columns 16-30
X projection center XC	in columns 31-45
Y projection center YC	in columns 46-60

The next card will contain 3 numbers (format 3 E15.8)

$F^2$ = focal length squared	in columns 1-15
Period of revolution of shutter	in columns 16-30
Number of occultations per revolution	in columns 31-45



The last card for station A will have two numbers (format 2I5)

The first number will be in columns 1-5

- = 1      if card output is desired
- = 0      if cards are not desired

The second number will be in columns 6-10

- = 1      if counterclockwise revolution of Super-Schmidt camera
- = -1     if clockwise revolution of Super-Schmidt camera
- = 0      if non-Super-Schmidt camera

This completes the input cards for station A. A second set in the same formats starting with card 3 will be required for station B.

#### ACKNOWLEDGEMENT

The author wishes to express her appreciation to Dr. R. E. McCrosky, Mrs. Annette Posen and Mr. G. Davoren of the Harvard Observatory who supplied training in the astrometric techniques discussed in this paper and who supplied a copy of the original Harvard Meteor Program and to Mr. E. Weston and Mr. J. Knight who supplied the list of star catalogs and charts for Appendix A. She especially wishes to thank Dr. H. L. Kasnitz and Mr. G. M. Shannon of Lincoln Laboratory who reviewed the paper and offered many helpful suggestions.

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3-21-6094

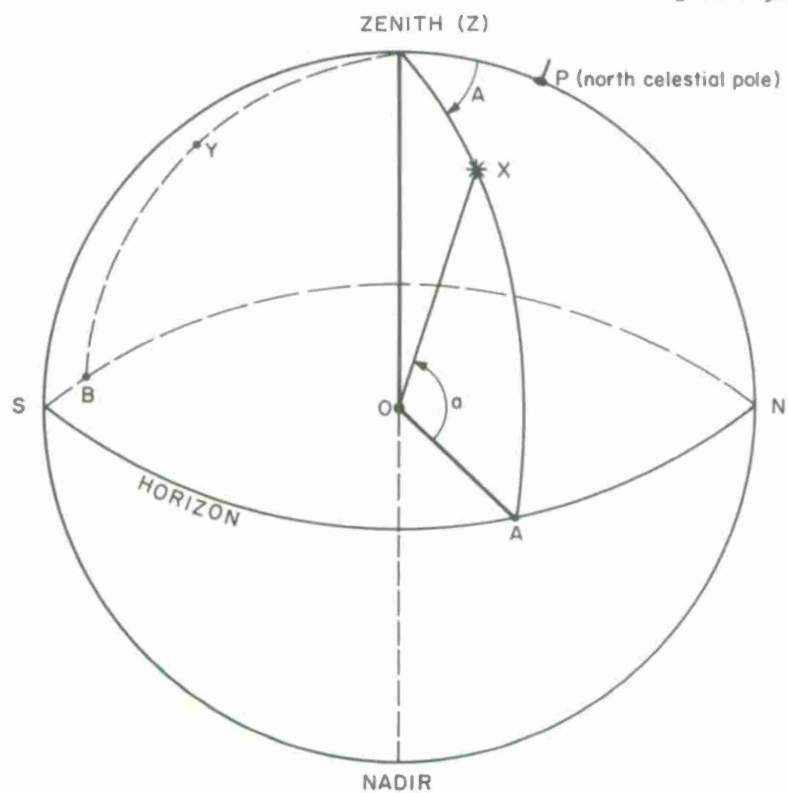


Fig. 1a. The altitude-azimuth system.

3-21-6095

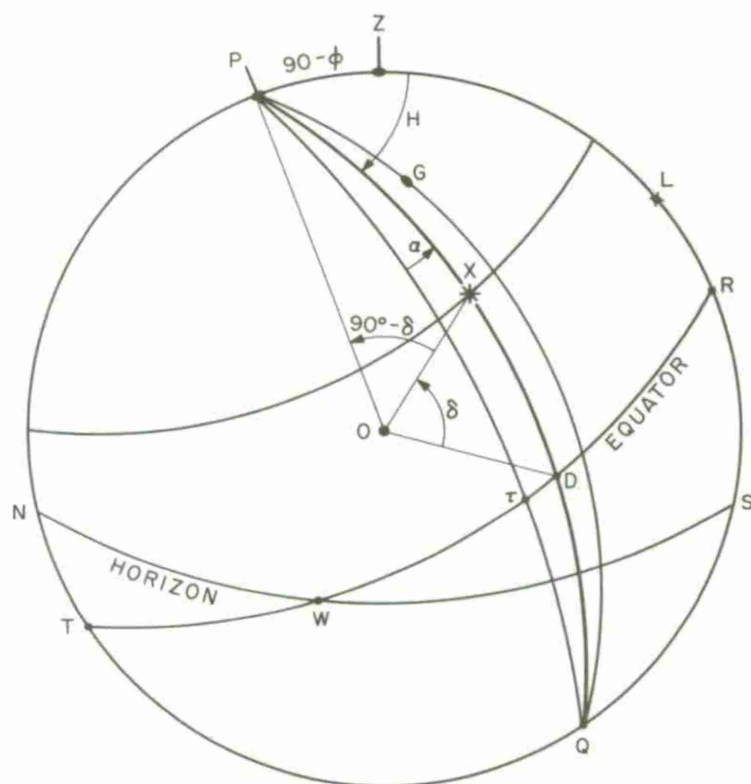


Fig. 1b. The declination-hour angle and the declination-right ascension systems.



3-21-6096

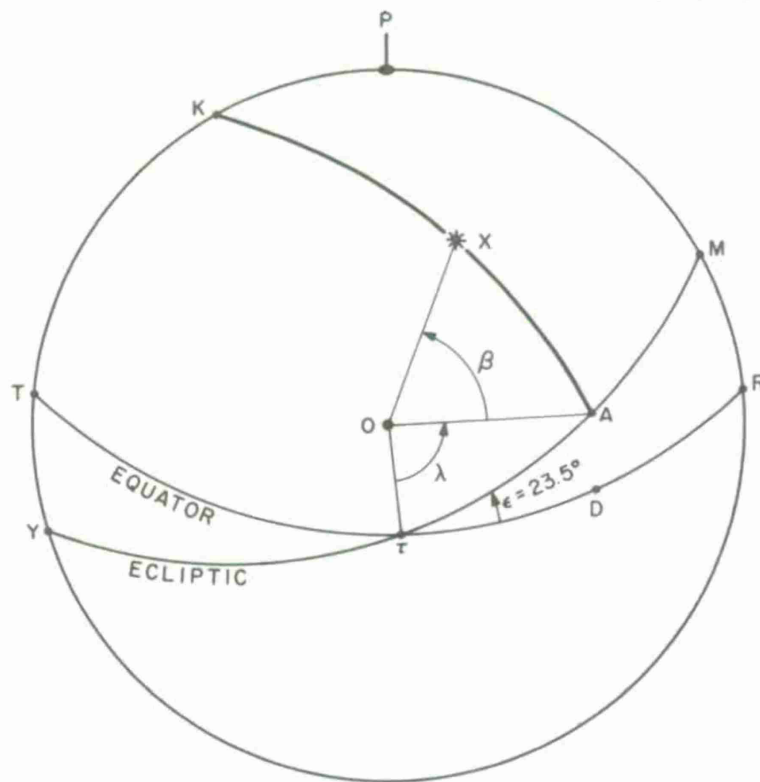


Fig. 1c. The celestial latitude-longitude system.

3-21-6097

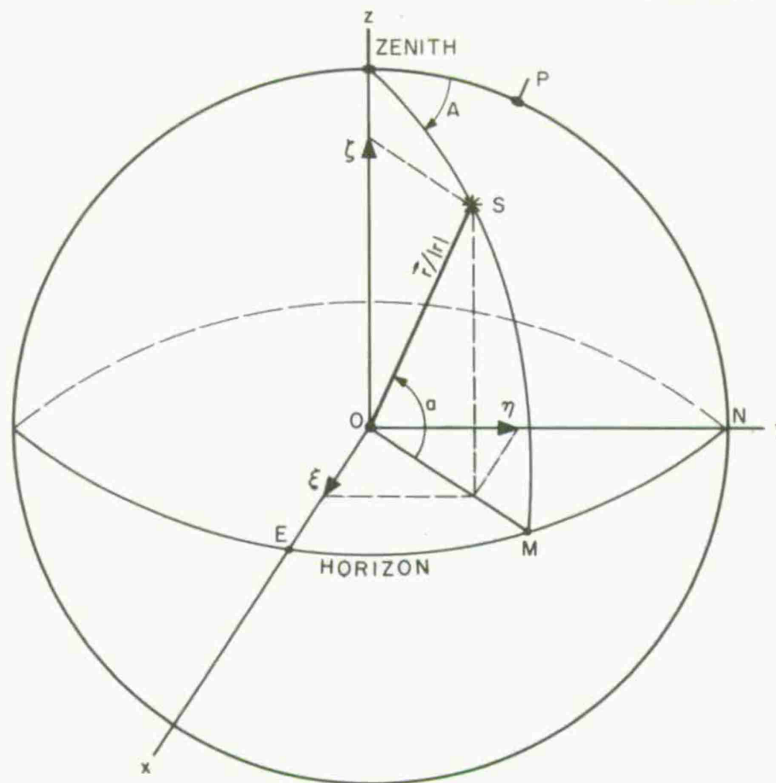


Fig. 2a. The altitude-azimuth system.

3-21-6098

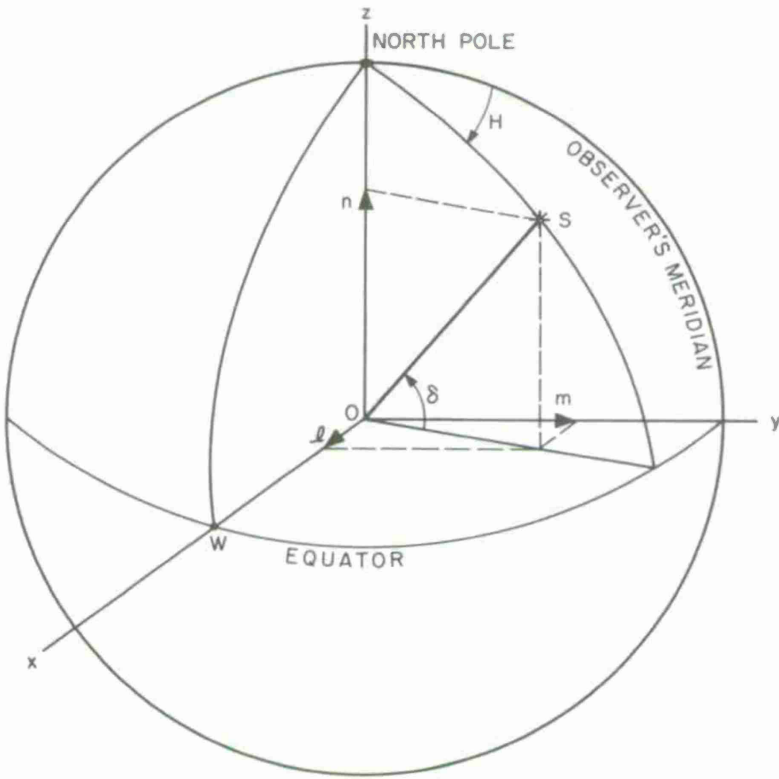


Fig. 2b. The declination-hour angle system.

3-21-6099

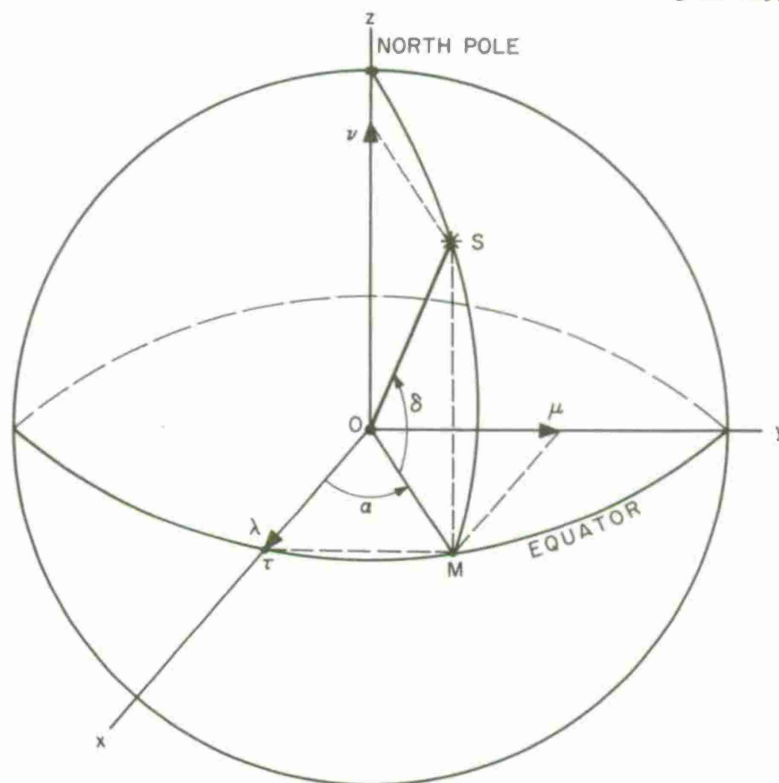


Fig. 2c. The declination-right ascension system.

3-21-6100

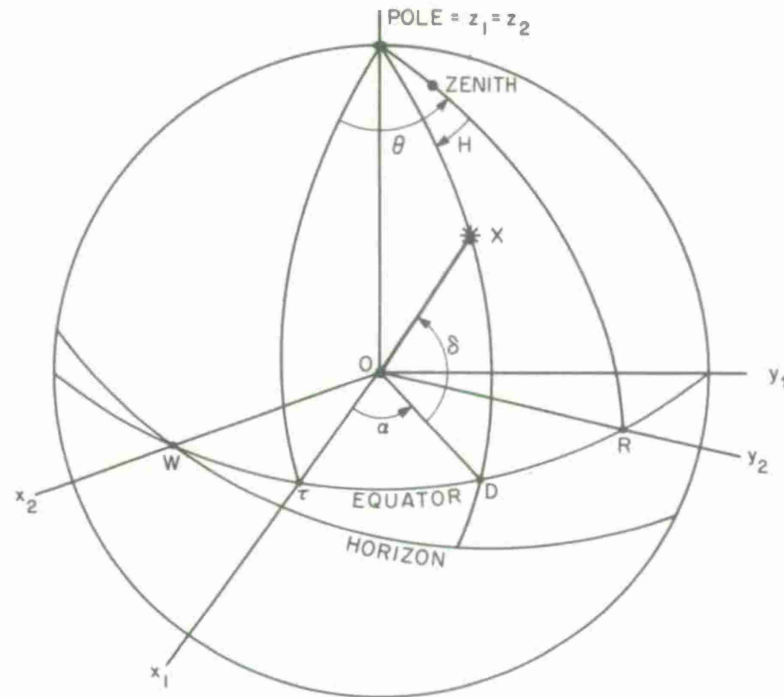


Fig. 3a. Transformation from the right ascension-declination system to the hour angle-declination system.

3-21-6101

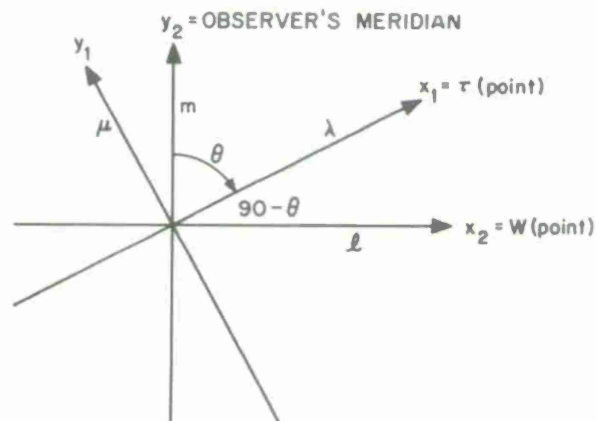


Fig. 3b. Transformation from the right ascension-declination system to the hour angle-declination system.



3-21-6102

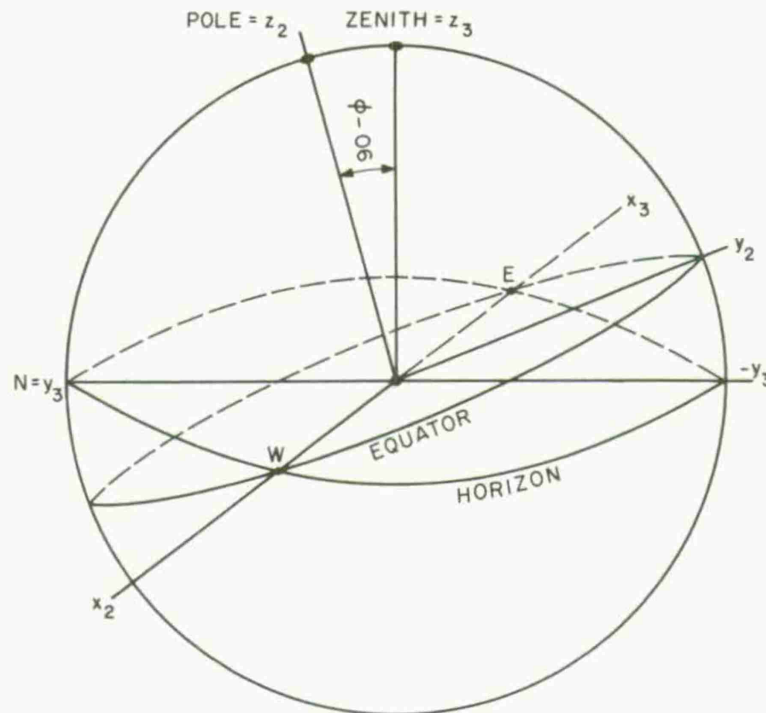


Fig. 3c. Transformation from the hour angle-declination system to the altitude-azimuth system.

3-21-6103

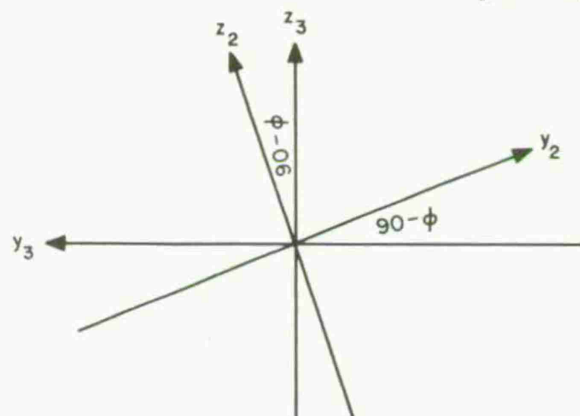


Fig. 3d. Transformation from the hour angle-declination system to the altitude-azimuth system.

3-21-6104

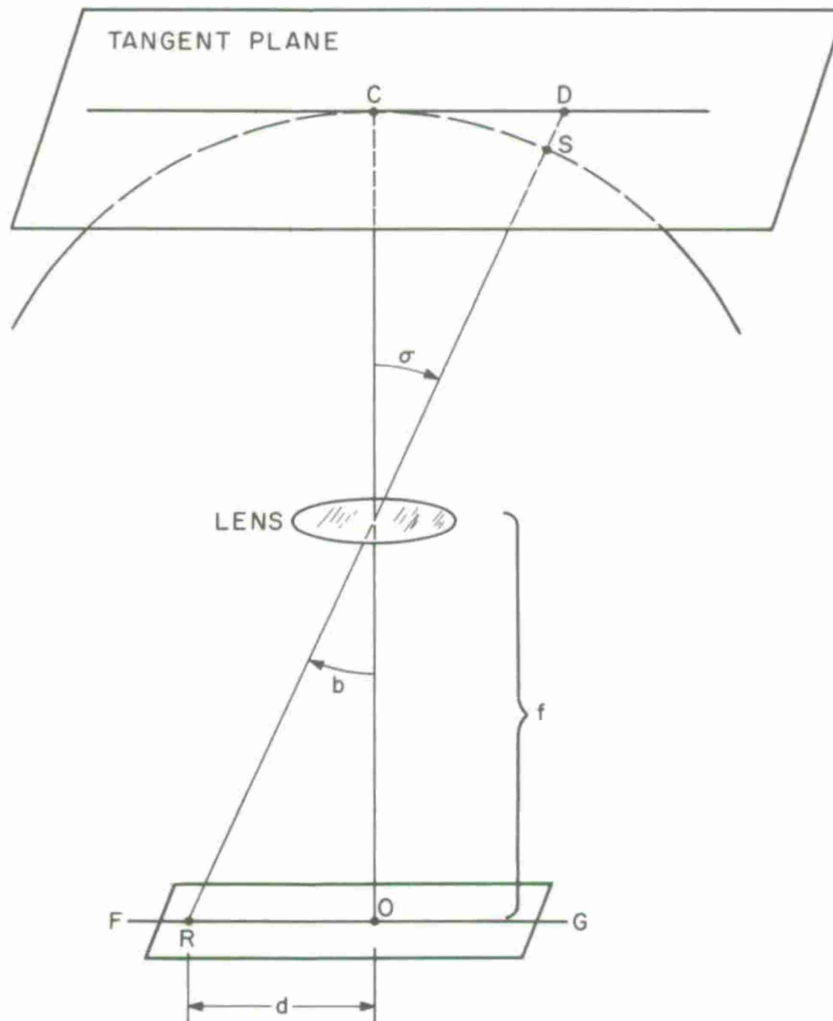


Fig. 4. Diagram of the optical system used for photographing celestial objects.

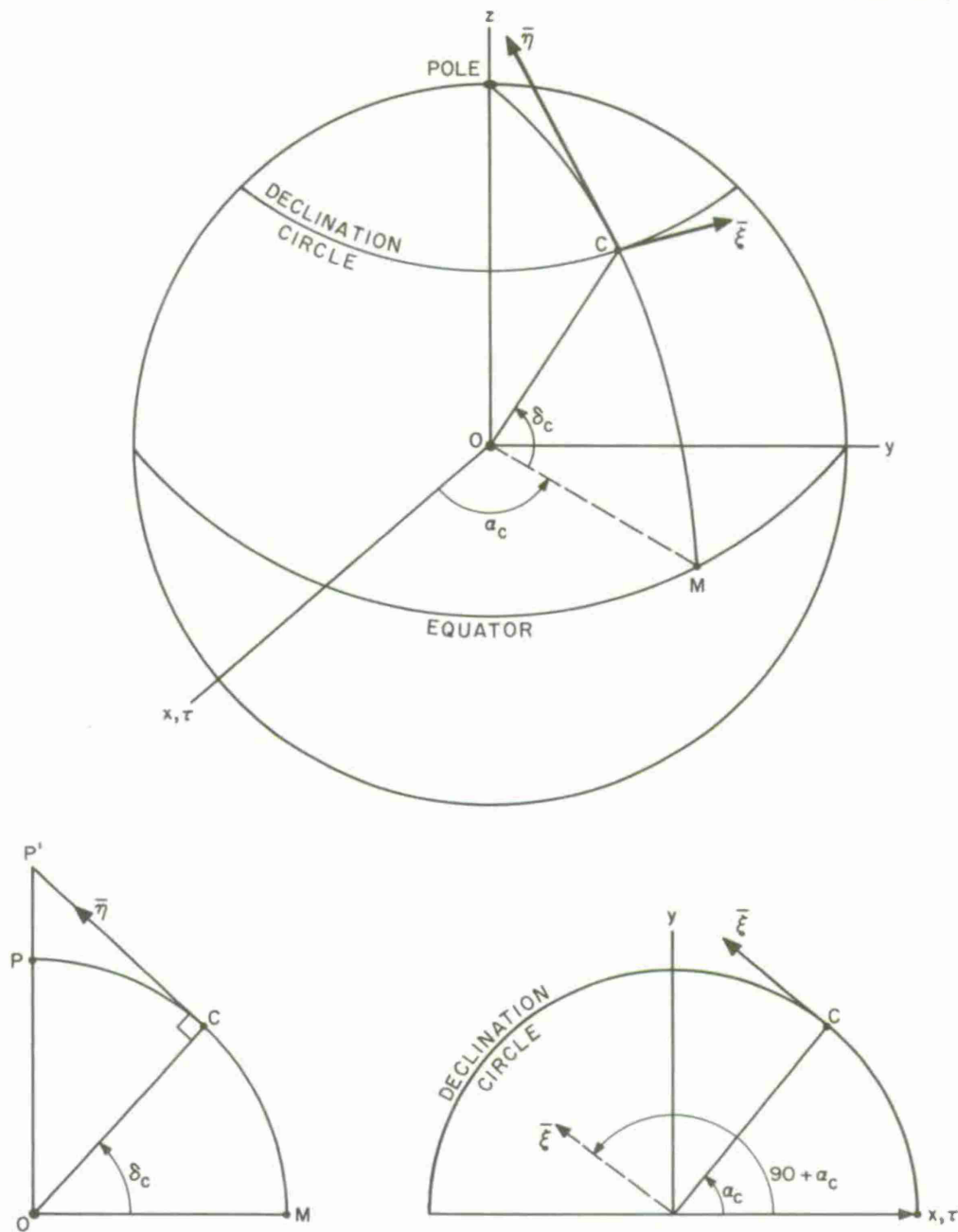


Fig. 5. Standard plate coordinate system with center at the optical center of the plate.

3-21-6107

	$x$	$y$	$z$
$\bar{\eta}$	$\lambda_{\eta}$	$\mu_{\eta}$	$\nu_{\eta}$
$\bar{\xi}$	$\lambda_{\xi}$	$\mu_{\xi}$	$0$
$z'$	$\lambda_c$	$\mu_c$	$\nu_c$

61

3-21-6108

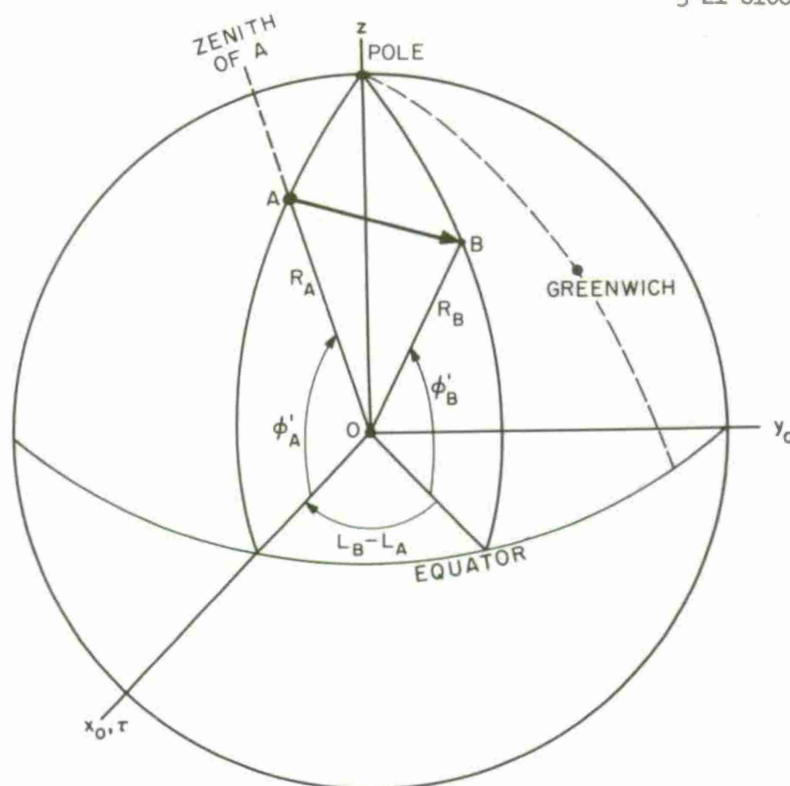


Fig. 7a. Relative coordinates of station B with respect to station A at sidereal time zero at station A.



3-21-6109

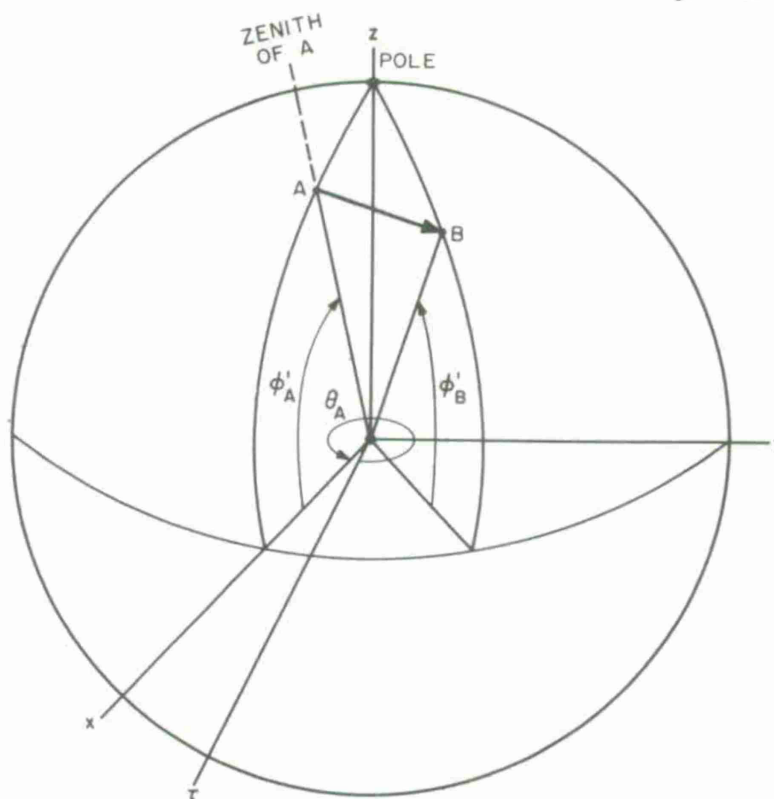


Fig. 7b. Relative coordinates of station B with respect to station A at sidereal time of the event.

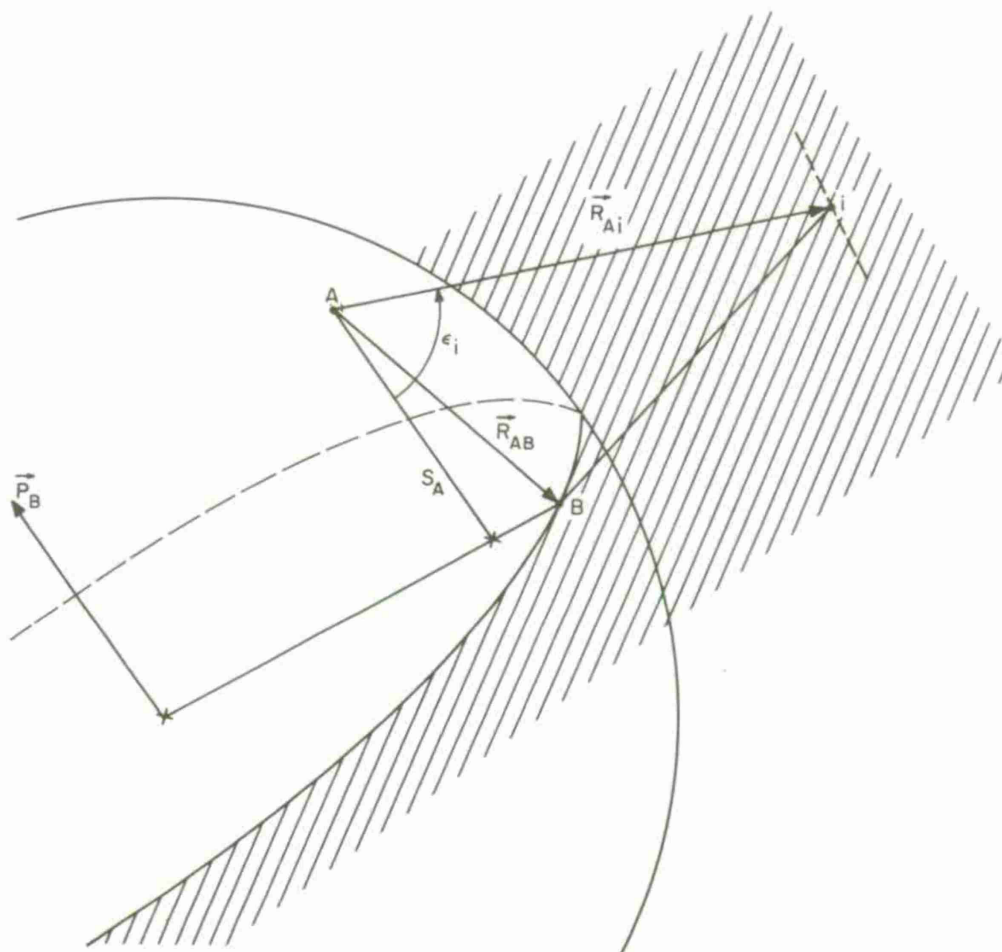


Fig. 8. Construction to show the plane through the missile trail and station B and the range vector to measured points from station A.

3-21-6111

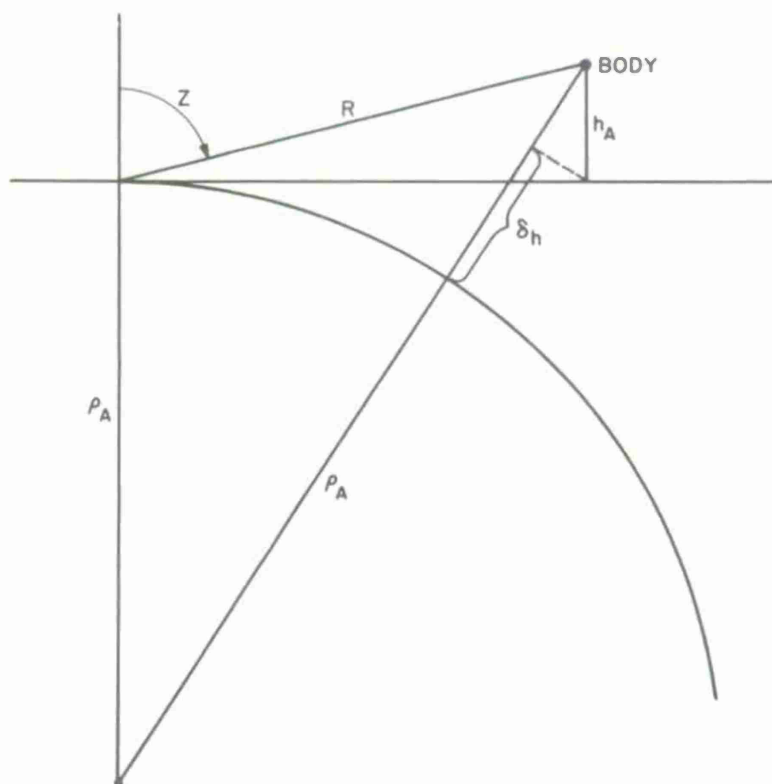


Fig. 9. Construction to show the method of computing height above mean sea level.

3-21-6112

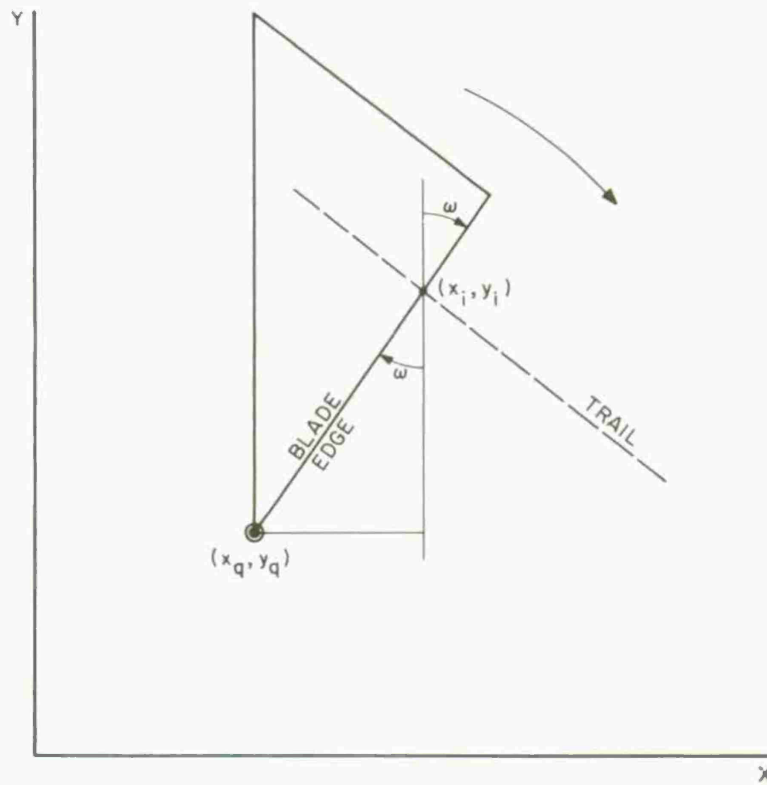


Fig. 10. The effect of the rotation of the shutter on points on the photographic plate.

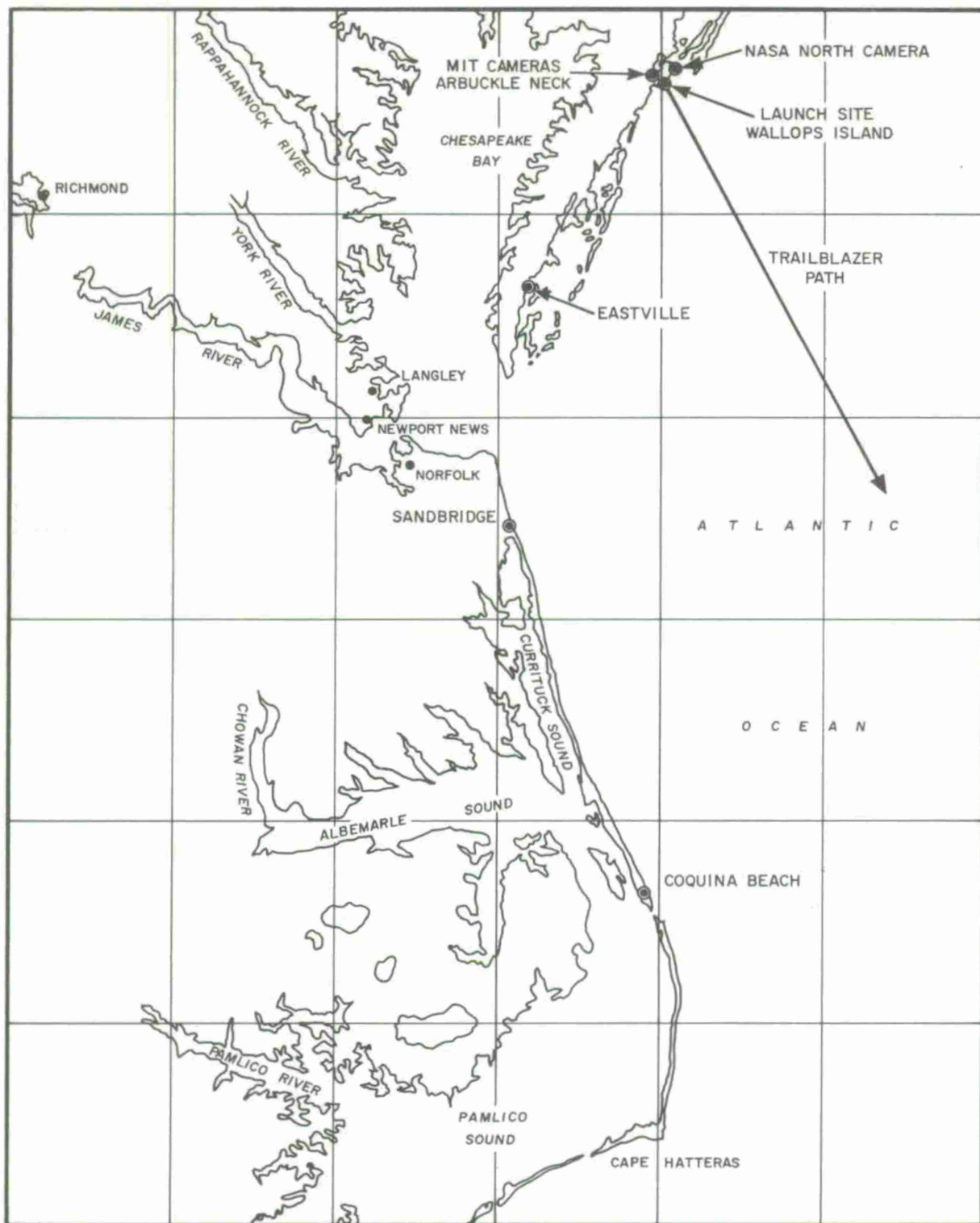


Fig. 11. Camera locations in relation to the launch site and the predicted trajectory of the missile.



P436-430

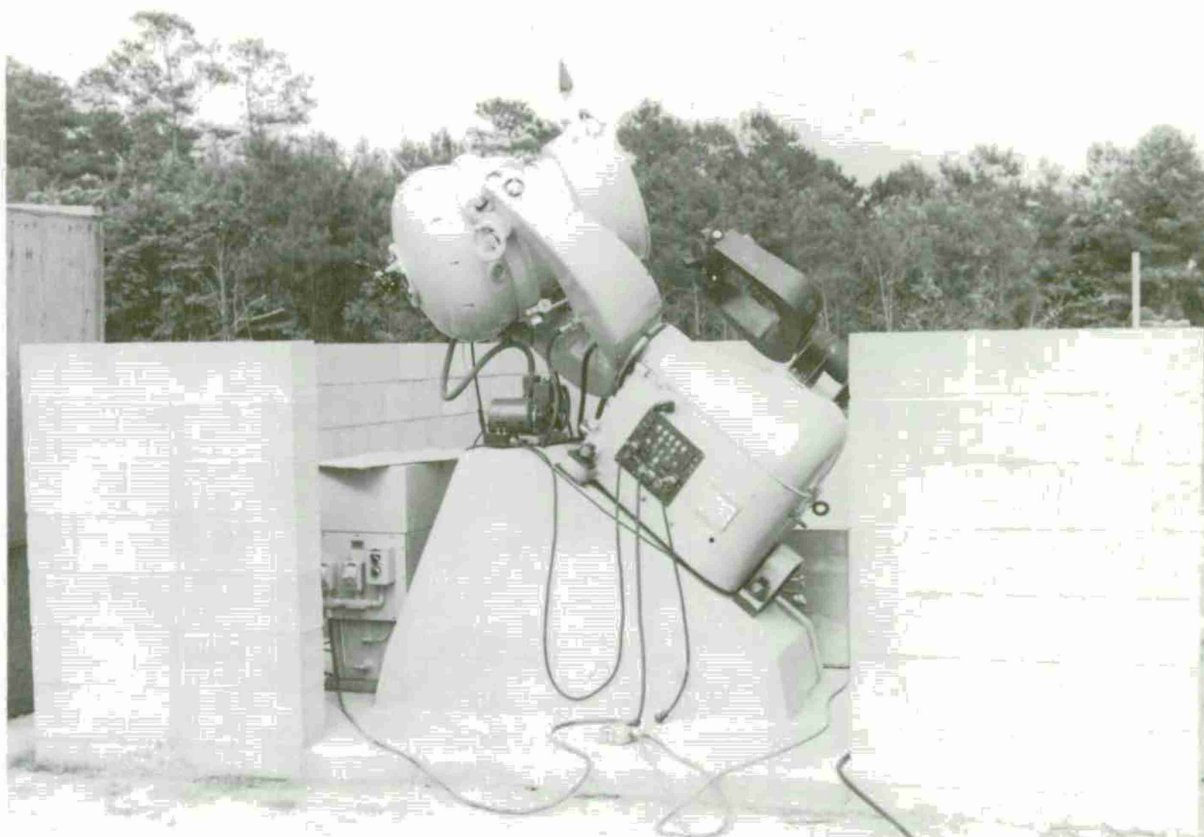


Fig. 12a. The Harvard Super-Schmidt camera at Arbuckle Neck, Virginia. The small camera on the right is a baby Schmidt.

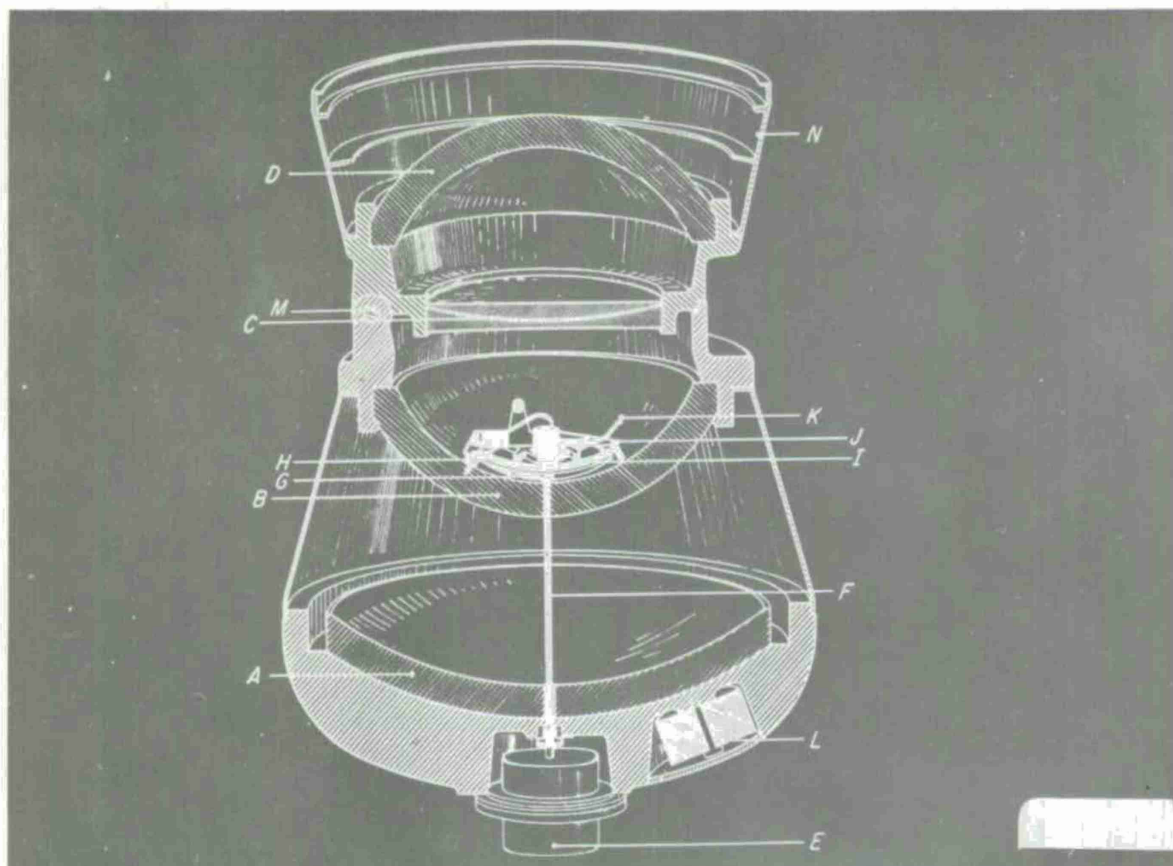


Fig. 12b. The Super-Schmidt optical system (from "Meteor Science and Engineering," Donald McKinley).

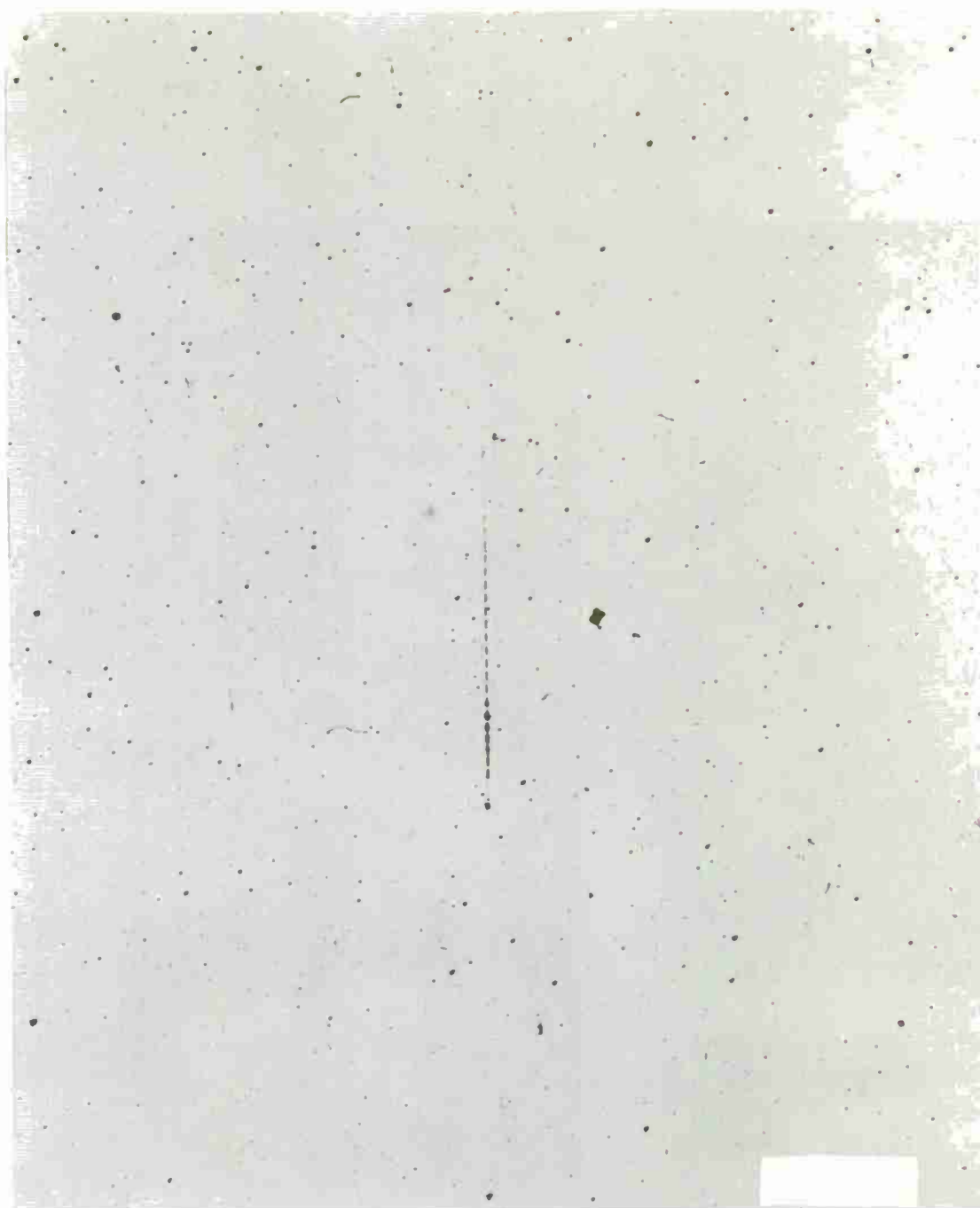


Fig. 13a. Eastville Super-Schmidt photograph of Trailblazer 1k sixth stage re-entry. Chopping rate, 15 chops per second.



Fig. 13b. Coquina Beach K 24 aerial camera photograph of Trailblazer Ic sixth stage re-entry. No chopping shutter.

C21-265



Fig. 13c. Eastville WILD BC 4 photograph of Trailblazer in sixth stage re-entry. No chopping shutter



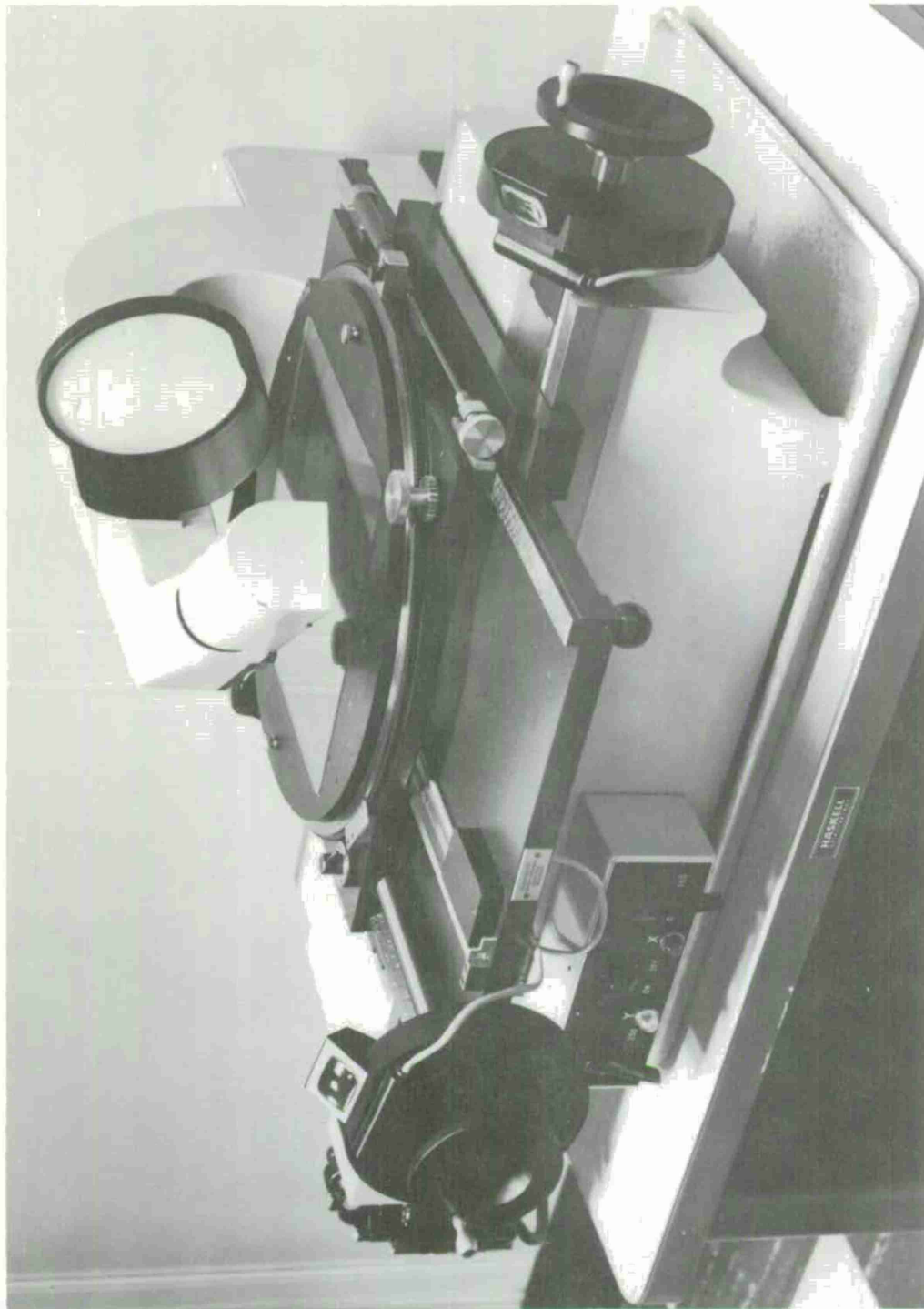
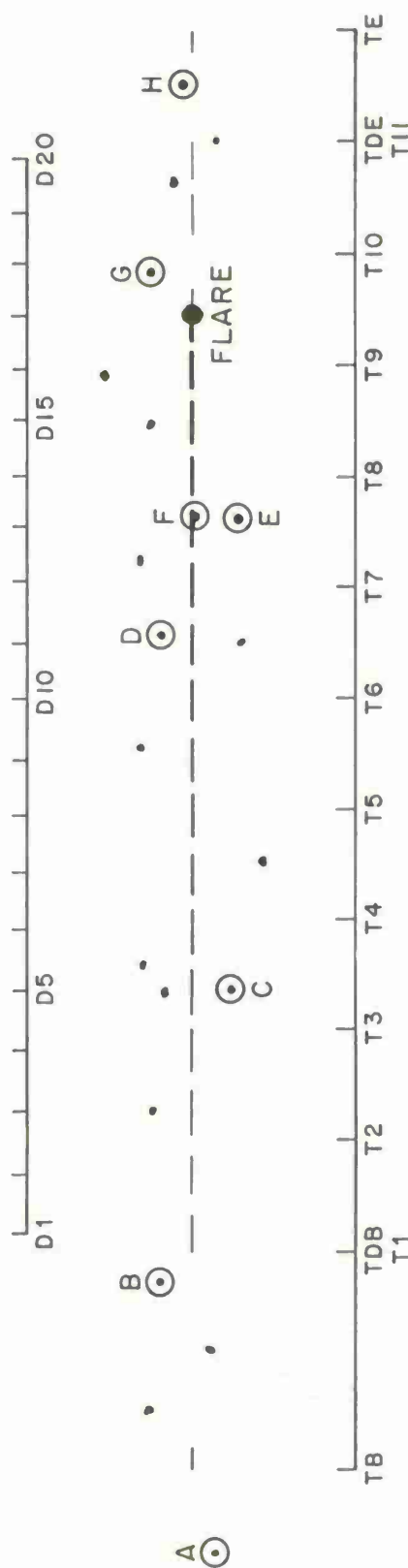


Fig. 14 A 2 screw rotatable horizontal measuring engine of a type suitable for reading ballistic plates.

## DASH CENTERS



## TYPICAL TRAIL-READING LOCATIONS

Fig. 15. Diagrammatic sketch of measurements to be made on a photographic plate.

3-21-6114

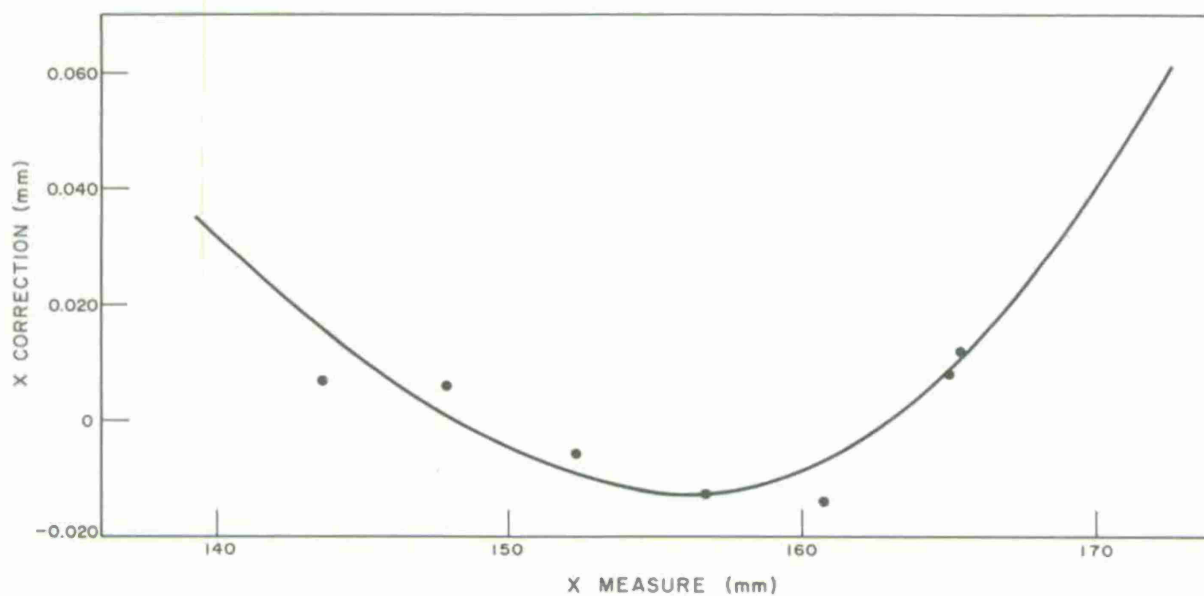


Fig. 16a. Sample curves showing residuals in X measurements.

3-21-6115

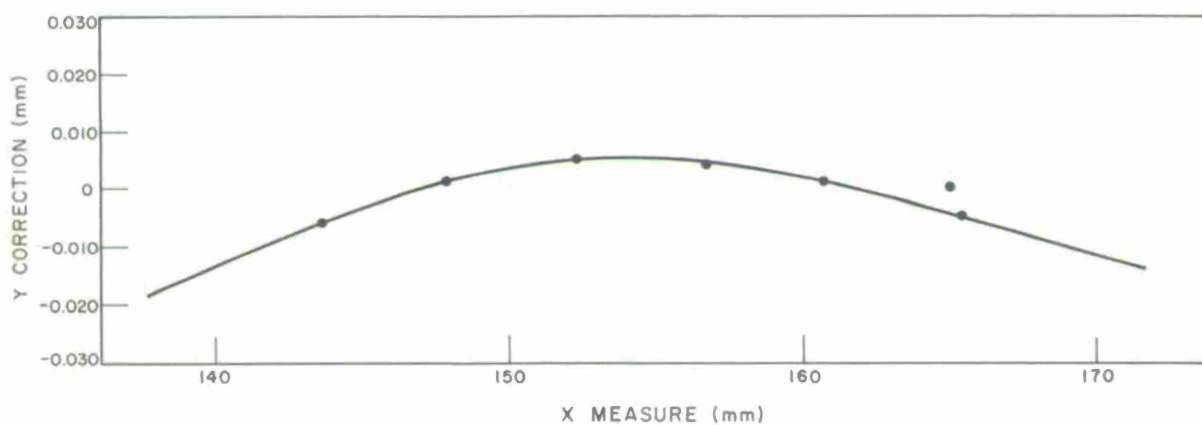


Fig. 16b. Sample curves showing residuals in Y measurements

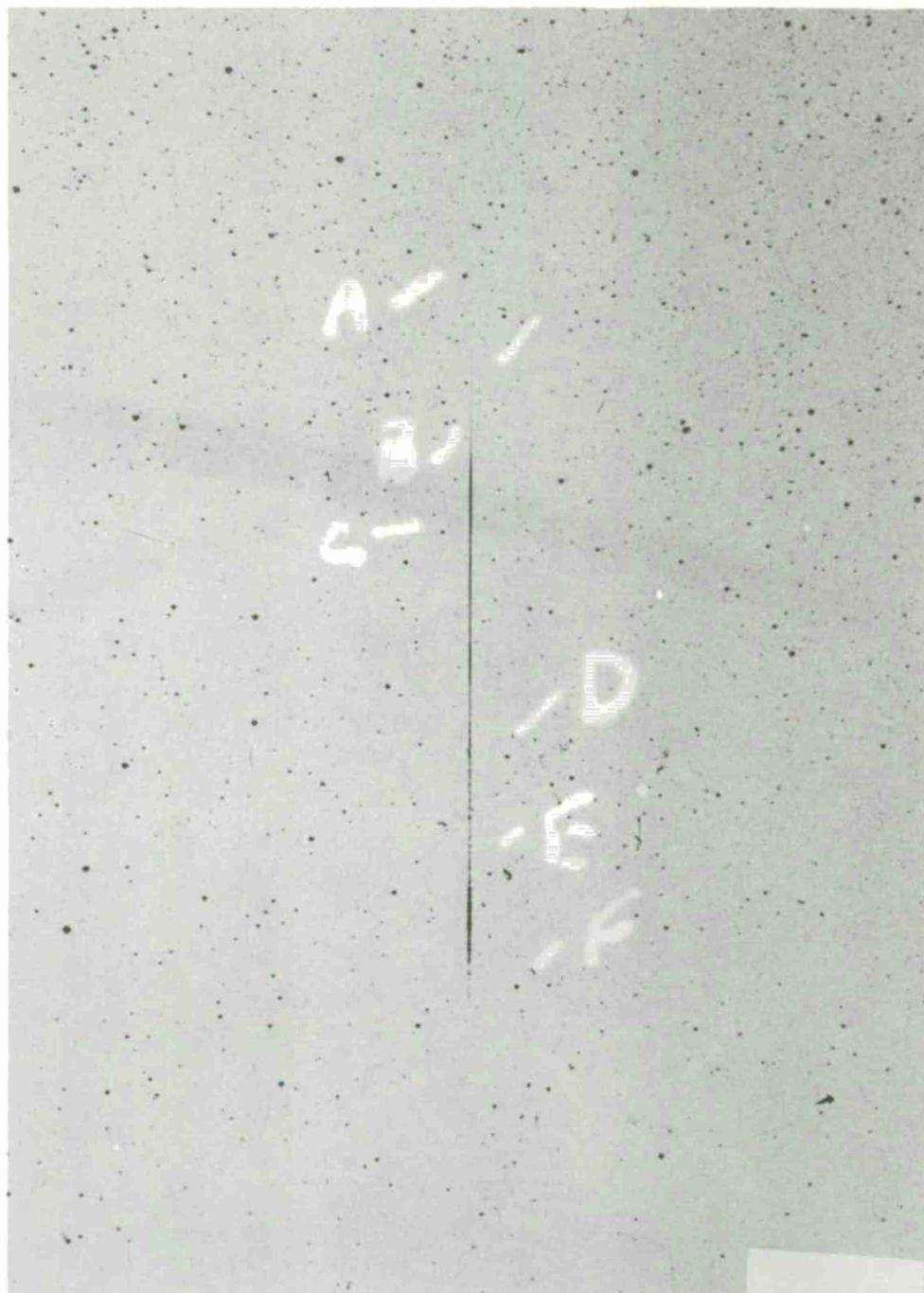


Fig. 17a. Arbuckle Neck Super-Schmidt photograph of Trailblazer Ik sixth stage re-entry. Chopping rate, 20 chops per second.

C33-851A



Fig. 17b. Eastville Super-Schmidt photograph of Trailblazer Ik sixth stage re-entry. Chopping rate, 20 chops per second.

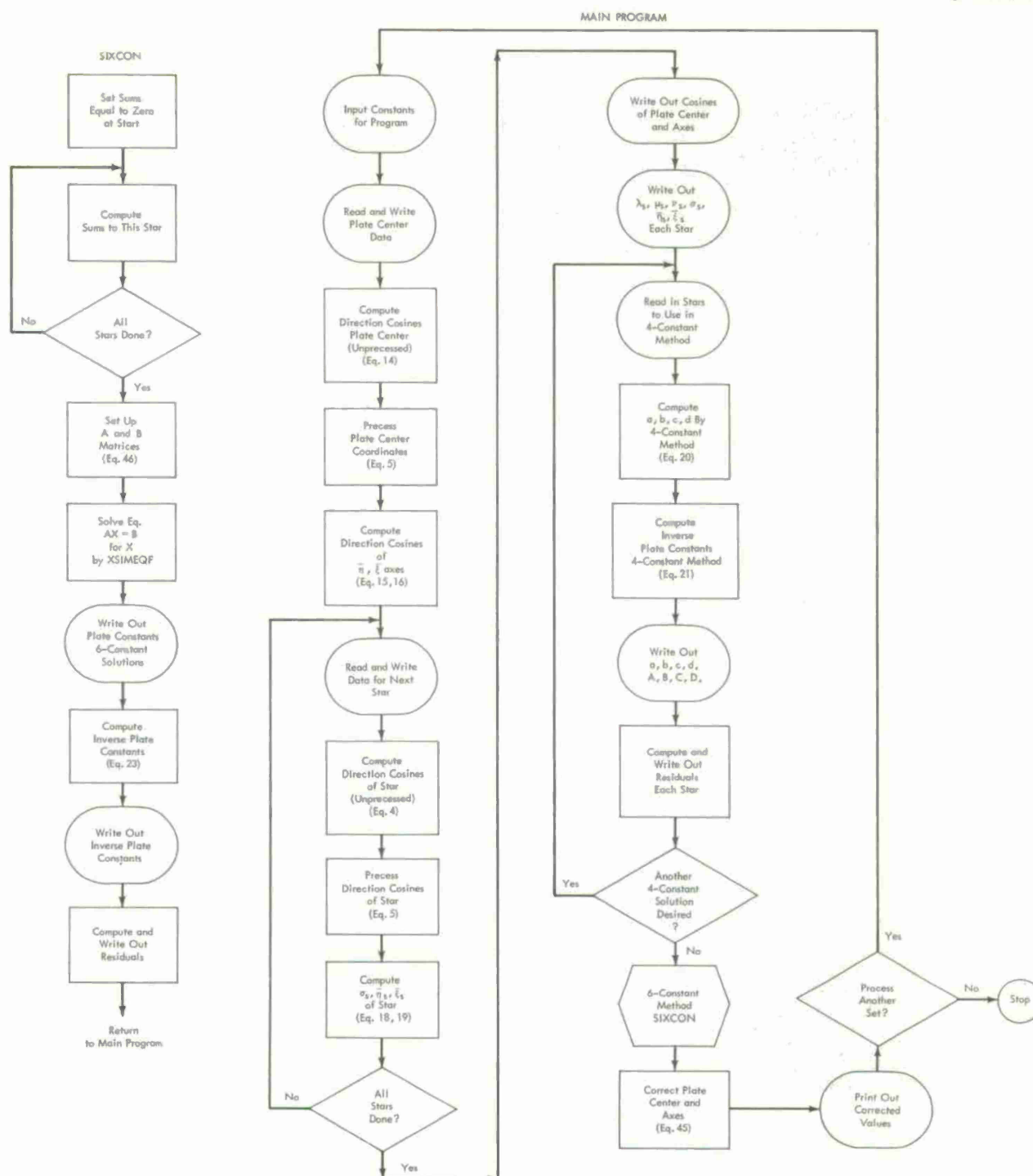


Fig. 18. Flow chart of the plate calibration program.



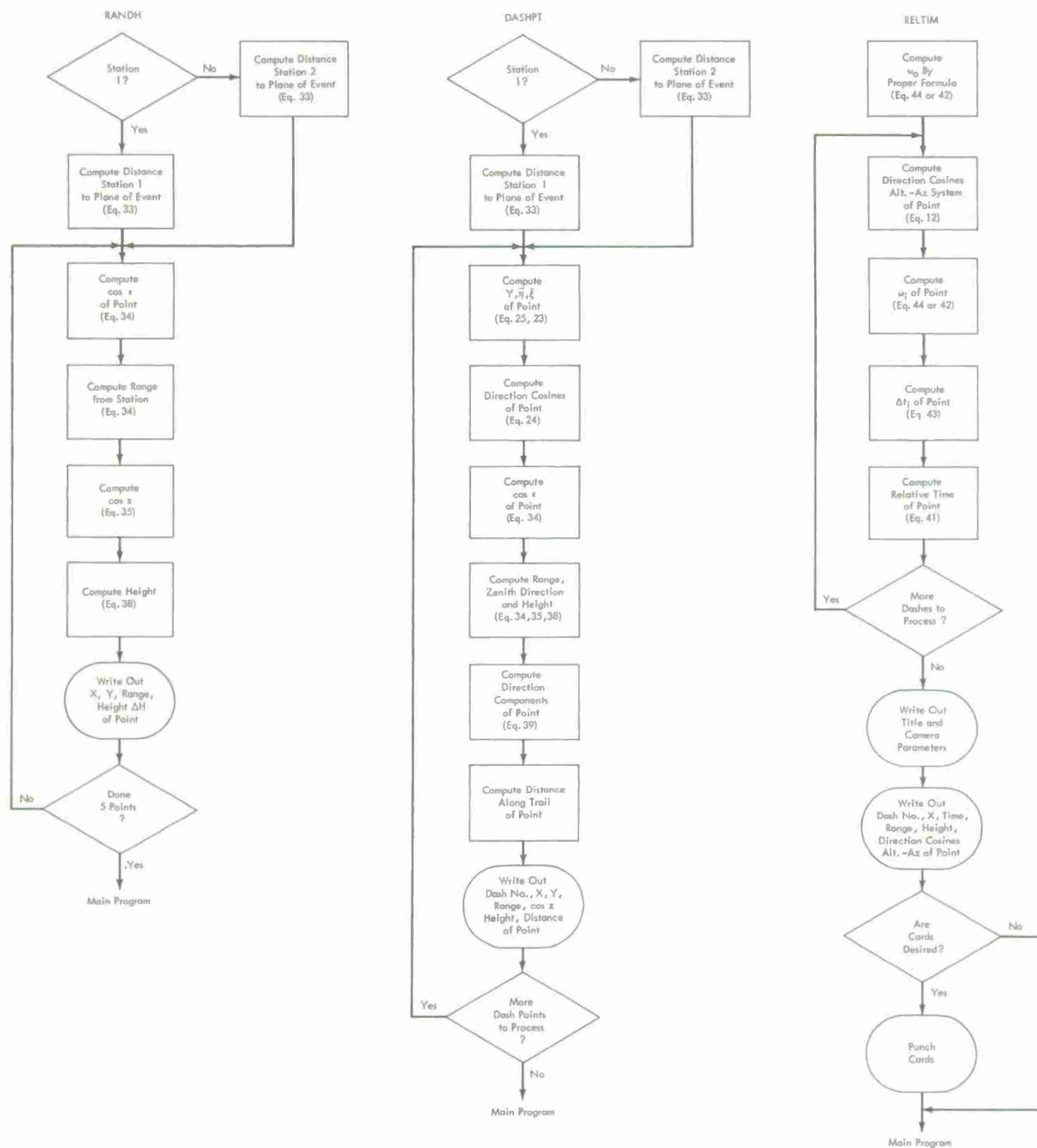


Fig. 19a. Flow chart of the optical trajectory program.

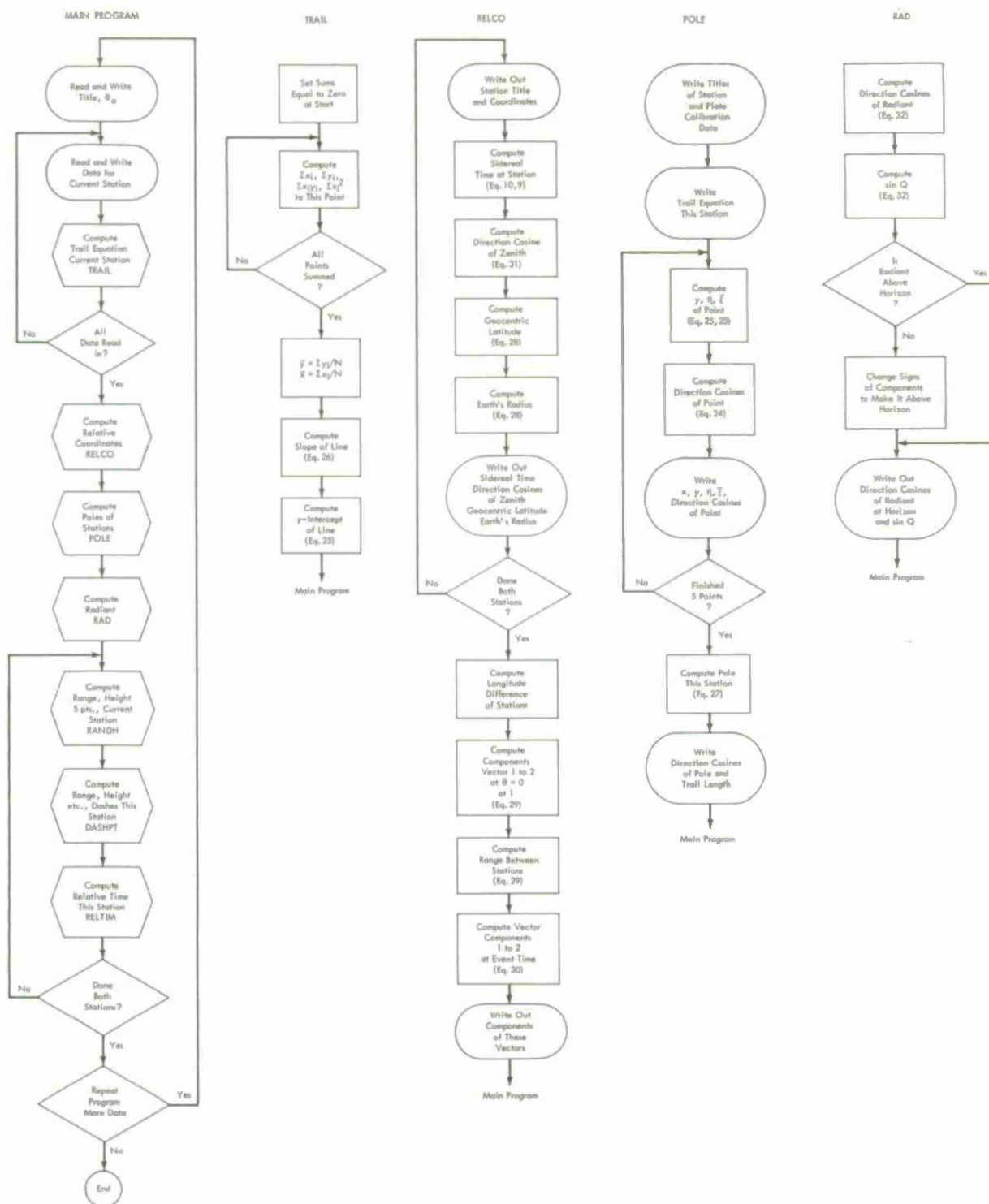


Fig. 19b. Flow chart of the optical trajectory program.

## APPENDIX A

The following charts and catalogues may be used to obtain the astronomical coordinates of the stars chosen for the plate calibration.

### Charts

Bonner 'Durchmusterung (B D)	+90° - 23°
Cordoba Resultados (C D)	-23° - -90°
Norton's Star Atlas very preliminary identification of brightest stars	

### Preliminary Catalogues

Bonner Durchmusterung (B D)	- 2° - +90°
Schurnfeld's Southern Extension of Bonner Durchmusterung (BD)	- 2° - -23°
Cordoba Durchmusterung (C D) used mainly -23° - -52°	-23° - -90°
Cape Photographic Durchmusterung (C P D) used mainly -52° to -90°	-40° - -90°

### Intermediate Catalogues

Henry Draper Catalogue intermediate catalogues used with the  
Boss General Catalogue

### Final or Precision Catalogues

AGK <sub>2</sub> with EBL Nord (for proper motion) (Zweiter Katalog Der Astronomischen Gesellschaft)	-90° - - 2°
Yale Zone Catalogues (Transactions of the Astronomical Observatory of Yale University)	+90° - +85° +60° - +50° +30° - -30°
Cape Zone Catalogues appearing in Cape Annals	-30° - -40°
Cape Zone Catalogue with Cape Proper Motion Catalogue or Cape Catalogue of Faint Stars	-52° - -64° -40° - -52°
La Plata Catalogues (or Cordoba Catalogues)	-64° - -82°
Boss General Catalogue	-90° - +90°

1TRAILBLAZER 1K SL PLATE

2 6  
01855-1963

0.99965377E+00-0.24128620E-01-0.10494900E-01  
0.24128620E-01+0.99970885E+00-0.12665000E-03  
0.10494900E-01-0.12661000E-03+0.99994491E+00

01950-1963

0.99999498E+00-0.29055300E-02-0.12631600E-02  
0.29055300E-02+0.99999578E+00-0.18400000E-05  
0.12631600E-02-0.18400000E-05+0.99999920E+00

19.+45.+00.0000-18.-17.-00.000  
20.+06.+13.7540-28.-35.-13.950+173.10150+020.41100  
20.+07.+28.9680-29.-54.-35.970+178.15500+020.73800  
20.+08.+17.2300-30.-56.-52.600+182.08600+021.12550  
20.+13.+45.2300-32.-45.-45.200+189.98750+018.83850  
20.+14.+25.7800-33.-50.-22.200+194.17100+019.44950  
20.+17.+02.5700-34.-44.-34.300+198.17700+018.45050

1  
2  
2  
2  
2  
2  
2

PC  
A  
B  
C  
D  
E  
F

1  
1 6

TRAILBLAZER IN SL PLATE

1855-1963	-0.02412862	0.02412862	0.99978885	-0.00012664	0.0104990	-0.00012661	0.9994490
0J9965376							

1958-1963	1964-1969	1970-1975	1976-1981	1982-1987	1988-1993	1994-1999	2000-2005	2006-2011	2012-2017	2018-2023
8,399,994,948	-1,082,905,53	-0,001,263,16	0,082,905,53	0,999,995,78	-0,000,000,00	0,001,263,16	-0,000,000,00	0,000,000,00	0,000,000,00	0,000,000,00

RT. ASC. CENTER= 19. H 45. M 0. S  
DEC. CENTER= 18. 0-17. M -0. S  
XC= -0. YC= -0.

RIGHT ASCENSION		DECLINATION		X	Y
HR	MIN	SEC	DES	MIN	SEC
20	6	13.7542	-35	-13.950	173.101
20	7	28.9693	-29	-35.970	178.155
20	8	17.2308	-39	-56	182.886
20	13	45.2300	-35	-45.200	189.987
20	14	25.7802	-33	-50	194.171
20	17	2.5709	-34	-44	198.177
20	20				20.411
20	20				20.738
20	20				21.125
20	20				18.838
20	20				15.449
20	20				18.450

## TRAILBLAZER 1K SL PLAYB

RT. ASC. CENTER= 19. H 45. M 0. S  
DEC. CENTER=-18. D-17. M -3. S

EANC=	4.4365523E-01	MUC=	-8.4117501E-01	MUC=	-3.8918370E-01
LANETA=	1.4423831E-01	MUETA=	-2.7347735E-01	MUETA=	-9.5182232E-01
LANEX1=	8.8451413E-01	MUEX1=	4.6651330E-01	MUEX1=	0

	LAMDA	MU	NU	COS5IG	ETA	KI
1	4.642323E-01	-7.4589619E-01	-4.7791556E-01	9.8116449E-01	-1.8707982E-01	6.1658682E-02
2	4.6944536E-01	-7.3479748E-01	-4.9805502E-01	9.7636391E-01	-2.1128492E-01	6.5038591E-02
3	4.7451467E-01	-7.2541849E-01	-5.1368488E-01	9.7202533E-01	-2.394131E-01	6.8724664E-02
4	4.8111595E-01	-7.0979596E-01	-5.4895716E-01	9.5326651E-01	-2.511877E-01	8.8817451E-02
5	4.8447468E-01	-6.9842597E-01	-5.628589E-01	9.5793892E-01	-2.8552329E-01	9.0767314E-01
6	4.6528436E-01	-6.7778937E-01	-5.6933831E-01	9.5258614E-01	-3.0833339E-01	1.0009910E-01



TRAIBLAZER IK SU PLATE  
A= 4.9899776E-01 B= -1.9947841E-02 C= 1.3275589E-02 D= 2.3524998E-01  
ACAP= 1.1634350E-03 BCAP= -4.7267051E-03 CCAP= -4.3256809E-02 DCAP= 6.5486677E-01  
STARS 1 6

AEXI= 1.1634350E-03 BEXI= -4.7267051E-03 CEXI= -4.3256809E-02  
AETA= -4.7267051E-03 BETA= -1.1634350E-03 CETA= 6.5486677E-01

NO	X(S)	Y(S)	DX	DY
1	173.101	20.411	-0.00000	-0.00000
2	178.165	20.738	-0.01006	-0.01223
3	182.086	21.125	0.01012	0.00454
4	189.987	18.838	0.01258	-0.01388
5	194.171	19.449	-0.00313	-0.01154
6	190.177	18.450	-0.00000	-0.00000

TRAIBLAZER IK SE PLATE  
MATRIX= 1

AX= 4.9968118E-01 BX= -1.9949473E-02 CX= 1.3276051E-02  
AY= -1.987777E-02 BY= -4.8090620E-01 CY= 2.3526559E-01

AEXI= 1.1634350E-03 BEXI= -4.7443160E-03 CEXI= -4.2743869E-02  
AETA= -4.7272654E-03 BETA= -1.1645432E-03 CETA= 6.5499187E-01

NO	X(S)	Y(S)	DX	DY
1	173.101	20.411	-0.00024	0.00567
2	178.165	20.738	-0.01040	-0.00855
3	182.086	21.125	0.00966	0.00610
4	189.987	18.838	0.01003	-0.00549
5	194.171	19.449	-0.00559	-0.00605
6	190.177	18.450	-0.00341	0.00031

TRAIBLAZER IK SL PLATE  
CORRECTED COORDINATES OF CENTER

LAMC= 4.4365523E-01 MUC= -0.4117500E-01 NUC= -3.0918370E-01  
LAMEA= 1.4427031E-01 MUEA= -2.7347735E-01 NUEA= 9.5100232E-01  
LAMEK= 8.8451413E-01 MUEK= 4.8651329E-01 NUEK= 0.



## 1TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

20.+20.+43.0380

OSL PLATE ARBUCKLE NECK

02.+56.+21.583 +37.+51.+23.2660+05.+02.+02.7830+00.0000000

15

0.17600000E+03+0.20140000E+02  
 0.00000000E+00+0.00000000E+00+0.15000000E+01+0.10000000E-02  
 0.30000000E+01+0.80000000E-02+0.45000000E+01+0.20000000E-02  
 0.60000000E+01+0.60000000E-02+0.75000000E+01+0.20000000E-02  
 0.90000000E+01+0.10000000E-02+0.10500000E+02+0.00000000E+00  
 0.12000000E+02+0.40000000E-02+0.13500000E+02+0.20000000E-02  
 0.15000000E+02+0.00000000E+00+0.16500000E+02+0.10000000E-02  
 0.18000000E+02+0.30000000E-02+0.19500000E+02+0.00000000E+00  
 0.21000000E+02+0.40000000E-02  
 0.11627049E-02-0.47443160E-02-0.42743869E-01  
 0.47272654E-02-0.11645432E-02+0.65499187E+00  
 0.44365523E+00+0.88451413E+00+0.14423830E+00  
 0.84117500E+00+0.46651330E+00-0.27347735E+00  
 0.30918370E+00+0.95100233E+00  
 175.86100+187.74500+194.74700+197.73400+198.64400

89

0	1+175.86100	1	4+175.94600	2	4+176.24500
3	1+176.53400	4	2+176.79600	5	1+177.05700
6	2+177.33200	7	4+177.59900	8	4+177.90000
9	3+178.16400	10	4+178.44300	11	4+178.72800

## TITLE

COOR A  
 N TR A  
 XO YO A  
 TR1 A  
 TR2 A  
 TR3 A  
 TR4 A  
 TR5 A  
 TR6 A  
 TR7 A  
 TR8 A  
 EXIS A  
 ETAS A  
 LAMB A  
 MU A  
 NU A  
 5 XS A

DASH1 A  
 DASH2 A  
 DASH3 A  
 DASH4 A

DATA MISSING

87 1+198.56700 0 1+198.64400

0.13320500E+03+0.23520000E+02+0.13320500E+03+0.23500000E+02  
 0.40740000E+04+0.10000000E+00+0.20000000E+01

0- 1

OSS PLATE EASTVILLE

02.+56.+21.583 +37.+20.+46.4300+05.+03.+36.7650+00.0000000

15

0.18000000E+03+0.44885000E+02  
 0.00000000E+00+0.20000000E-01+0.15000000E+01+0.40000000E-02  
 0.30000000E+01+0.10000000E-02+0.45000000E+01+0.10000000E-02  
 0.60000000E+01+0.50000000E-02+0.75000000E+01+0.80000000E-02  
 0.90000000E+01+0.90000000E-02+0.10500000E+02+0.10000000E-02  
 0.12000000E+02+0.50000000E-02+0.13500000E+02+0.13000000E-01  
 0.15000000E+02+0.70000000E-02+0.16500000E+02+0.60000000E-02  
 0.18000000E+02+0.30000000E-02+0.19500000E+02+0.50000000E-02  
 0.20100000E+02+0.10000000E-02  
 0.25734744E-02-0.41302408E-02-0.28662772 +00  
 0.41605029E-02-0.25337586E-02+0.54358602 +00  
 0.75910489E+00+0.65095623E+00+0.39516512 -02  
 0.65094741E+00+0.75911517E+00-0.33886189 -02  
 0.52056018E-02+0.99998645E+00  
 176.84700+186.71700+199.26900+202.44800+202.52500

51

0	1+176.84700	1	1+186.71700	2	1+186.97500
3	1+187.36800	4	1+187.66000	5	1+187.96200

DASH30 A

CAM2 A

TITLE B  
 COOR B  
 N TR B  
 XO YO B  
 TR1 B  
 TR2 B  
 TR3 B  
 TR4 B  
 TR5 B  
 TR6 B  
 TR7 B  
 TR8 B  
 EXIS B  
 ETAS B  
 LAMB B  
 MU B  
 NU B  
 5 XS B

DASH1 B  
 DASH2 B

DATA MISSING

52 1+202.22000 53 1+202.44800 0 1+202.52500

0.12665600E+03+0.10333000E+02+0.12665600E+03+0.10333000E+02  
 0.40740000E+04+0.10000000E+00+0.20000000E+01

0+ 1

DASH17 B  
 CAM1 B  
 CAM2 B

# TRAILBLAZER 1K OPTICAL TRAJECTORY S1-S3

TIME: 28.0M 28.0M 43.03000

SL PLATE ARBUCKLE NECK

TIME: 21.0M 56.0M 21.58000

PHI: 37.0D 51.0M 23.26000

RANGE: 5.0H 2.0M 2.78300

ELEV: 21.0M KFI

0.13760000E 03 0.20140000E 02

0.15000000E 01 0.99999999E-03  
0.30000000E 01 0.79999999E-02  
0.59999999E 01 0.59999999E-02  
0.89999999E 01 0.99999999E-03  
0.12000000E 02 0.39999999E-02  
0.15000000E 02 0.39999999E-02  
0.18000000E 02 0.30000000E-02  
0.20999999E 02 0.39999999E-02  
0.11627040E-02 0.47443160E-02  
0.47272654E-02 0.11645432E-02  
0.44365522E-00 0.88451412E 00  
0.44117508E 00 0.46651330E-00  
0.30910369E-00 0.95100233E 00  
175J06100 187.74500 194.74700

197.73400 198.64400  
0 1 175.86100 1 4 175.94600 2 4 176.24500  
3 1 176.53400 4 2 176.79600 5 1 177.25700

07 1 198.56700 0 1 198.64400

0.13320500E 03 0.23520000E 02 0.13320500E 03 0.23520000E 02  
0.16740000E 04 0.89999999E-03 0.20000000E 01

-2

SS PLATE EASTVILLE

TIME: 21.0M 56.0M 21.58300

PHI: 37.0D 20.0M 46.43000

RANGE: 5.0H 3.0M 30.76500

ELEV: 21.0M KFI

0.18000000E 03 0.44840999E 02

0.20000000E-01 0.15000000E 01 0.39999999E-02  
0.59999999E 01 0.49999999E-02 0.74999999E 01 0.79999999E-02  
0.89999999E 01 0.89999999E-02 0.10500000E 02 0.99999999E-03  
0.30000000E 01 0.99999999E-03 0.45000000E 01 0.99999999E-03  
0.12000000E 02 0.49999999E-02 0.13500000E 02 0.13000000E-01  
0.15000000E 02 0.70000000E-02 0.16500000E 02 0.59999999E-02  
0.10000000E 02 0.30000000E-02 0.19499999E 02 0.49999999E-02  
0.20100000E 02 0.99999999E-03  
0.25734743E-02 0.41302400E-02 0.20662772E-00  
0.41605099E-02 0.25337566E-02 0.54358602E 00  
0.75910499E 00 0.65095623E 00 0.39516512E-02  
0.6594741E 00 0.75911517E 00 0.33886189E-02  
0.52056017E-02 0.9999845E 00  
176J04700 186.71700 199.26900

202.44000 202.52500  
0 1 176.64700 1 1 186.71700 2 1 186.97500  
3 1 187.36800 4 1 187.66000 5 1 187.96200

52 1.202.0200 53 1.202.4400 0 1.202.5200  
 0.1266500E 03 0.1033299E 02 0.1266500E 03 0.1033299E 02  
 0.4074000E 04 0.0999999E 00 0.2000000E 01

# TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

SL PLATE ARBUCKLE NECK  
 TIME(1)= 21.0 56.0 21.50305  
 PHIZ(1)= 37.0 51.0 23.26605  
 LONG(1)= 5.0 2.0 2.78305  
 ELEV(1)= 0.0 KFT  
 THETA(1)= 5.556506E 02 DEG  
 ELZ(1)= -7.453305E 01 ENZ(1)= -2.656172E 01 ENZ(1)= 6.136854E 01  
 PHIPRIME(1)= 3.746935E 01 DEG  
 RHOT(1)= 2.0900031E 04 KFT

SS PLATE EASTVILLE  
 TIME(2)= 21.0 56.0 21.50305  
 PHIZ(2)= 37.0 51.0 23.26605  
 LONG(2)= 5.0 2.0 2.78305  
 ELEV(2)= 0.0 KFT  
 THETA(2)= 5.5926706E 02 DEG  
 ELZ(2)= -7.505780E 01 ENZ(2)= -2.623222E 01 ENZ(2)= 6.0663003E 01  
 PHIPRIME(2)= 3.7160012E 01 DEG  
 RHOT(2)= 2.0900030E 04 KFT  
 EXIAB= 1.1297827E 02 KFT ETAB= -1.1384110E 02 KFT  
 EXIAB= -1.4469111E 02 KFT ETAB= 6.9190042E 01 KFT ZETAB= -1.4720337E 02 KFT  
 ERROR= 0.

# TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

SL PLATE ARBUCKLE NECK  
 CAX= 1.1627049E-03 CBX= -4.7443100E-03 CCX= -4.2743060E-02  
 CAY= -4.7272654E-03 CBY= -1.1645432E-03 CCY= 6.5499107E-01  
 ELC= 4.4365522E-01 ELEXI= 8.8451412E-01 ELETA= 1.4423830E-01  
 EMC= -4.4117500E-01 EMEXI= 4.6651329E-01 ENETA= -2.7347735E-01  
 ENC= -3.0918370E-01 ENETA= 9.5108232E-01  
 Y= -2.7142855E-04 X + 2.0191687E 01  

X	Y	EL	EM	EN	ERROR
175.00100	20.14395	4.6320002E-01	-7.3946494E-01	-4.8649866E-01	-1.1175871E-07
187.74500	20.14073	4.6119066E-01	-7.080274E-01	-5.3376189E-01	-8.1956387E-08
194.77800	20.13883	4.5940169E-01	-6.9816676E-01	-5.5912415E-01	-1.1175871E-07
197.73400	20.13802	4.5851187E-01	-6.8211490E-01	-5.6963675E-01	-9.6857548E-08
198.66400	20.13777	4.5822631E-01	-6.7965131E-01	-5.7280245E-01	-1.0430813E-07

ELP= 0.0591900E-01 EMP= 4.0138974E-01 ENP= 2.3244799E-01  
 SINL= 1.0334059E-01  
 TRAIL LENGTH= 5.9320299E 00 DEG  
 ERROR= 0. 1.8626451E-09 -3.7252903E-09



# TRAIRBLAZER 1K OPTICAL TRAJECTORY SL-SS

## SL PLATE CENTERVILLE

CAY 215734744E-03 CCX -4.1302407E-03 CCY -2.8662772E-01  
 CAY -4.11605029E-03 CCY -2.8337586E-03 CCY 5.4358602E-01  
 EL 7.5910408E-01 ELEXI 6.5095623E-01 ELSTA 3.9516512E-03  
 EN 6.5094740E-01 ENEXI 7.5911517E-01 ENBTA -3.3886189E-03  
 EN 5.512656817E-03 ENETA 9.9998645E-01

Y = -4.6962025E-04 X + 4.4972580E 01

X	Y	EL	EM	EN	ERROR
176J04700	44.88945	7.1411376E-01	-6.3367878E-01	-2.9747698E-01	-1.6391277E-07
186J07100	44.88401	7.2108868E-01	-6.8775406E-01	-3.3270045E-01	-1.6391277E-07
196J06900	44.87892	7.2778268E-01	-5.7418778E-01	-3.7529845E-01	-1.5646219E-07
202J44800	44.87743	7.2980918E-01	-5.6551241E-01	-2.8566981E-01	-1.3411045E-07
202J52500	44.87739	7.2983903E-01	-5.6530397E-01	-2.8591888E-01	-1.4156103E-07

ELR 6.7034609E-01 ENR 5.2145012E-01 ENP 5.1761764E-01

SIN 1.1260283E-01  
 TRAIL LENGTH 624653798E 00 DEG

ERROR 0. 9.3132257E-09 9.3132257E-09

# TRAIRBLAZER 1K OPTICAL TRAJECTORY SL-SS

## RADIANT

ELR 2J3644279E-01 ENR -8.2194099E-01 ENR 5.1817740E-01  
 SIN 3J6006961E-01  
 ERROR -7.4505806E-09

# TRAIRBLAZER 1K OPTICAL TRAJECTORY SL-SS

## SL PLATE ARBUCKLE NECK

X	Y	R	H	DH
175J06100	20.14395	4.2641687E 02	-1.8743069E 02	3.5095215E 00
187J74500	20.14073	4.1514006E 02	-1.9698637E 02	3.1945801E 00
194J74700	20.13883	4.0945571E 02	-2.0226319E 02	3.0319824E 00
197J73400	20.13802	4.072896E 02	-2.0444007E 02	2.9672852E 00
198J64400	20.13777	4.0657312E 02	-2.0509491E 02	2.9479980E 00

# TRAILBLAZER IK OPTICAL TRAJECTORY SL-SS

## 3L PRATE ARBUCKLE NECK

DASH	X	Y	R	COSZ	H	D
0	175.86100	20.14395	4.2641687E 02	-4.4777826E-01	-1.8743869E 02	0.
1	175.94600	20.14393	4.2632837E 02	-4.4803252E-01	-1.8759214E 02	1.9072623E-01
2	176.24500	20.14385	4.2681799E 02	-4.4892894E-01	-1.8775288E 02	8.6876735E-01
3	176.53400	20.14377	4.251939E 02	-4.4978818E-01	-1.879931E 02	1.5071833E 00
4	176.79600	20.14370	4.2544885E 02	-4.5056872E-01	-1.8821173E 02	2.0919503E 00
5	177.05700	20.14363	4.2518246E 02	-4.5134822E-01	-1.8842848E 02	2.6732273E 00
6	177.32200	20.14355	4.2490190E 02	-4.5216220E-01	-1.8865680E 02	3.2851821E 00
7	177.59900	20.14348	4.2463066E 02	-4.5295429E-01	-1.8887832E 02	3.8779456E 00
85	178.18700	20.13789	4.0698119E 02	-5.1050288E-01	-2.047663E 02	4.6553334E 01
86	178.42100	20.13783	4.0674006E 02	-5.1108939E-01	-2.0492741E 02	4.6988972E 01
87	178.56700	20.13779	4.0662822E 02	-5.1149741E-01	-2.0503933E 02	4.7296088E 01
0	178.64400	20.13777	4.0657312E 02	-5.1169864E-01	-2.0509491E 02	4.7439383E 01

NO CHECKS AT END OF DASHPT

DIVIDE CHECK AT DY/DX

# TRAILBLAZER IK OPTICAL TRAJECTORY SL-SS

## 3L PRATE ARBUCKLE NECK

XQ= 1.3320500E 02 YQ= 2.3520800E 01 XC= 1.3320500E 02 YC= 2.3520800E 01  
F2= 4.0740800E 03 P= 1.0000000E-01 OCCULT= 2.0000000E 00

DASH	X	TIME	RANGE	HEIGHT	ALPHA	BETA	GAMMA
0	175.861	0.	4.2641687E 02	-1.8743869E 02	8.5219424E-01	-2.7066493E-01	-4.4777826E-01
1	175.946	5.0002480E-02	4.2632837E 02	-1.8758214E 02	8.5190928E-01	-2.7088936E-01	-4.4803251E-01
2	176.245	1.0001113E-01	4.2601799E 02	-1.875280E 02	8.5126758E-01	-2.7107822E-01	-4.4892594E-01
3	176.534	1.5001937E-01	4.2571939E 02	-1.8799331E 02	8.5056069E-01	-2.723975E-01	-4.4978817E-01
4	176.796	2.0002676E-01	4.2544985E 02	-1.8821173E 02	8.4993417E-01	-2.7312933E-01	-4.5056871E-01
5	177.057	2.5003403E-01	4.2518246E 02	-1.8842848E 02	8.4930116E-01	-2.7381552E-01	-4.5134522E-01
6	177.332	3.0004159E-01	4.2490190E 02	-1.8865680E 02	8.4863318E-01	-2.7453768E-01	-4.5216219E-01
7	177.599	3.5004885E-01	4.2463066E 02	-1.8887832E 02	8.4790372E-01	-2.7523803E-01	-4.5295420E-01
85	178.187	4.2504293E 00	4.0698119E 02	-2.047663E 02	7.9540558E-01	-3.2667529E-01	-5.1050288E-01
86	178.421	4.3004321E 00	4.0674006E 02	-2.0492741E 02	7.9481066E-01	-3.2720565E-01	-5.1108939E-01
87	178.567	4.3504341E 00	4.0662822E 02	-2.0503933E 02	7.9439611E-01	-3.2757463E-01	-5.1149741E-01
0	178.644	4.3516058E-04	4.0657312E 02	-2.0509491E 02	7.9419141E-01	-3.2775663E-01	-5.1169864E-01



# TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

35 PLATE EASTVILLE

DASH	X	Y	R	H	OH
1	176.847	44.88945	4.3547645E 02	-2.3637424E 02	3.1999512E 00
2	186.975	44.88481	4.2398649E 02	-2.4454113E 02	2.8698730E 00
3	187.368	44.87892	4.1169173E 02	-2.5404575E 02	2.5187422E 00
4	187.668	44.87743	4.0094455E 02	-2.631151E 02	2.4209551E 00
5	187.962	44.87739	4.0887970E 02	-2.722461E 02	2.4272461E 00

# TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

36 PLATE EASTVILLE

DASH	X	Y	R	H	OH
1	176.847	44.88945	4.3547645E 02	-2.3637424E 02	3.1999512E 00
2	186.975	44.88481	4.2398649E 02	-2.4454113E 02	2.8698730E 00
3	187.368	44.88451	4.2378886E 02	-2.4474630E 02	2.8698730E 00
4	187.668	44.88437	4.2328005E 02	-2.4505740E 02	2.8698730E 00
5	187.962	44.88423	4.2297705E 02	-2.4528810E 02	2.8698730E 00
6	188.259	44.88407	4.2265689E 02	-2.4552613E 02	2.8698730E 00
7	188.517	44.88392	4.2239707E 02	-2.4576624E 02	2.8698730E 00
8	188.781	44.88378	4.2219475E 02	-2.4600493E 02	2.8698730E 00
9	189.045	44.88364	4.2200000E 02	-2.4624362E 02	2.8698730E 00
10	189.309	44.88350	4.2180475E 02	-2.4648231E 02	2.8698730E 00
11	189.573	44.88336	4.2160950E 02	-2.4672100E 02	2.8698730E 00
12	189.837	44.88322	4.2141425E 02	-2.4695969E 02	2.8698730E 00
13	190.101	44.88308	4.2121900E 02	-2.4719838E 02	2.8698730E 00
14	190.365	44.88294	4.2102375E 02	-2.4743707E 02	2.8698730E 00
15	190.629	44.88280	4.2082850E 02	-2.4767576E 02	2.8698730E 00
16	190.893	44.88266	4.2063325E 02	-2.4791445E 02	2.8698730E 00
17	191.157	44.88252	4.2043800E 02	-2.4815314E 02	2.8698730E 00
18	191.421	44.88238	4.2024275E 02	-2.4839183E 02	2.8698730E 00
19	191.685	44.88224	4.2004750E 02	-2.4863052E 02	2.8698730E 00
20	191.949	44.88210	4.1985225E 02	-2.4886921E 02	2.8698730E 00
21	192.213	44.88196	4.1965700E 02	-2.4910790E 02	2.8698730E 00
22	192.477	44.88182	4.1946175E 02	-2.4934659E 02	2.8698730E 00
23	192.741	44.88168	4.1926650E 02	-2.4958528E 02	2.8698730E 00
24	193.005	44.88154	4.1907125E 02	-2.4982397E 02	2.8698730E 00
25	193.269	44.88140	4.1887600E 02	-2.5006266E 02	2.8698730E 00
26	193.533	44.88126	4.1868075E 02	-2.5030135E 02	2.8698730E 00
27	193.797	44.88112	4.1848550E 02	-2.5054004E 02	2.8698730E 00
28	194.061	44.88098	4.1829025E 02	-2.5077873E 02	2.8698730E 00
29	194.325	44.88084	4.1809500E 02	-2.5101742E 02	2.8698730E 00
30	194.589	44.88070	4.1790000E 02	-2.5125611E 02	2.8698730E 00
31	194.853	44.88056	4.1770500E 02	-2.5149480E 02	2.8698730E 00
32	195.117	44.88042	4.1750975E 02	-2.5173349E 02	2.8698730E 00
33	195.381	44.88028	4.1731450E 02	-2.5197218E 02	2.8698730E 00
34	195.645	44.88014	4.1711925E 02	-2.5221087E 02	2.8698730E 00
35	195.909	44.88000	4.1692400E 02	-2.5244956E 02	2.8698730E 00
36	196.173	44.87986	4.1672875E 02	-2.5268825E 02	2.8698730E 00
37	196.437	44.87972	4.1653350E 02	-2.5292694E 02	2.8698730E 00
38	196.701	44.87958	4.1633825E 02	-2.5316563E 02	2.8698730E 00
39	196.965	44.87944	4.1614300E 02	-2.5340432E 02	2.8698730E 00
40	197.229	44.87930	4.1594775E 02	-2.5364301E 02	2.8698730E 00
41	197.493	44.87916	4.1575250E 02	-2.5388170E 02	2.8698730E 00
42	197.757	44.87902	4.1555725E 02	-2.5412039E 02	2.8698730E 00
43	198.021	44.87888	4.1536200E 02	-2.5435908E 02	2.8698730E 00
44	198.285	44.87874	4.1516675E 02	-2.5459777E 02	2.8698730E 00
45	198.549	44.87860	4.1497150E 02	-2.5483646E 02	2.8698730E 00
46	198.813	44.87846	4.1477625E 02	-2.5507515E 02	2.8698730E 00
47	199.077	44.87832	4.1458100E 02	-2.5531384E 02	2.8698730E 00
48	199.341	44.87818	4.1438575E 02	-2.5555253E 02	2.8698730E 00
49	199.605	44.87804	4.1419050E 02	-2.5579122E 02	2.8698730E 00
50	199.869	44.87790	4.1400000E 02	-2.5602991E 02	2.8698730E 00
51	200.133	44.87776	4.1380475E 02	-2.5626860E 02	2.8698730E 00
52	200.397	44.87762	4.1360950E 02	-2.5650729E 02	2.8698730E 00
53	200.661	44.87748	4.1341425E 02	-2.5674598E 02	2.8698730E 00

# TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

35 PLATE EASTVILLE

DASH	X	Y	R	H	OH
1	176.847	44.88945	4.3547645E 02	-2.3637424E 02	3.1999512E 00
2	186.975	44.88481	4.2398649E 02	-2.4454113E 02	2.8698730E 00
3	187.368	44.88451	4.2378886E 02	-2.4474630E 02	2.8698730E 00
4	187.668	44.88437	4.2328005E 02	-2.4505740E 02	2.8698730E 00
5	187.962	44.88423	4.2297705E 02	-2.4528810E 02	2.8698730E 00
6	188.259	44.88407	4.2265689E 02	-2.4552613E 02	2.8698730E 00
7	188.517	44.88392	4.2239707E 02	-2.4576624E 02	2.8698730E 00
8	188.781	44.88378	4.2219475E 02	-2.4600493E 02	2.8698730E 00
9	189.045	44.88364	4.2200000E 02	-2.4624362E 02	2.8698730E 00
10	189.309	44.88350	4.2180475E 02	-2.4648231E 02	2.8698730E 00
11	189.573	44.88336	4.2160950E 02	-2.4672100E 02	2.8698730E 00
12	189.837	44.88322	4.2141425E 02	-2.4695969E 02	2.8698730E 00
13	189.837	44.88322	4.2141425E 02	-2.4695969E 02	2.8698730E 00
14	190.101	44.88308	4.2121900E 02	-2.4719838E 02	2.8698730E 00
15	190.365	44.88294	4.2102375E 02	-2.4743707E 02	2.8698730E 00
16	190.629	44.88280	4.2082850E 02	-2.4767576E 02	2.8698730E 00
17	190.893	44.88266	4.2063325E 02	-2.4791445E 02	2.8698730E 00
18	191.157	44.88252	4.2043800E 02	-2.4815314E 02	2.8698730E 00
19	191.421	44.88238	4.2024275E 02	-2.4839183E 02	2.8698730E 00
20	191.685	44.88224	4.2004750E 02	-2.4863052E 02	2.8698730E 00
21	191.949	44.88210	4.1985225E 02	-2.4886921E 02	2.8698730E 00
22	192.213	44.88196	4.1965700E 02	-2.4910790E 02	2.8698730E 00
23	192.477	44.88182	4.1946175E 02	-2.4934659E 02	2.8698730E 00
24	192.741	44.88168	4.1926650E 02	-2.4958528E 02	2.8698730E 00
25	193.005	44.88154	4.1907125E 02	-2.4982397E 02	2.8698730E 00
26	193.269	44.88140	4.1887600E 02	-2.5006266E 02	2.8698730E 00
27	193.533	44.88126	4.1868075E 02	-2.5030135E 02	2.8698730E 00
28	193.797	44.88112	4.1848550E 02	-2.5054004E 02	2.8698730E 00
29	194.061	44.88098	4.1829025E 02	-2.5077873E 02	2.8698730E 00
30	194.325	44.88084	4.1809500E 02	-2.5101742E 02	2.8698730E 00
31	194.589	44.88070	4.1790000E 02	-2.5125611E 02	2.8698730E 00
32	194.853	44.88056	4.1770500E 02	-2.5149480E 02	2.8698730E 00
33	195.117	44.88042	4.1750975E 02	-2.5173349E 02	2.8698730E 00
34	195.381	44.88028	4.1731450E 02	-2.5197218E 02	2.8698730E 00
35	195.645	44.88014	4.1711925E 02	-2.5221087E 02	2.8698730E 00
36	195.909	44.88000	4.1692400E 02	-2.5244956E 02	2.8698730E 00
37	196.173	44.87986	4.1672875E 02	-2.5268825E 02	2.8698730E 00
38	196.437	44.87972	4.1653350E 02	-2.5292694E 02	2.8698730E 00
39	196.701	44.87958	4.1633825E 02	-2.5316563E 02	2.8698730E 00
40	196.965	44.87944	4.1614300E 02	-2.5340432E 02	2.8698730E 00
41	197.229	44.87930	4.1594775E 02	-2.5364301E 02	2.8698730E 00
42	197.493	44.87916	4.1575250E 02	-2.5388170E 02	2.8698730E 00
43	197.757	44.87902	4.1555725E 02	-2.5412039E 02	2.8698730E 00
44	198.021	44.87888	4.1536200E 02	-2.5435908E 02	2.8698730E 00
45	198.285	44.87874	4.1516675E 02	-2.5459777E 02	2.8698730E 00
46	198.549	44.87860	4.1497150E 02	-2.5483646E 02	2.8698730E 00
47	198.813	44.87846	4.1477625E 02	-2.5507515E 02	2.8698730E 00
48	199.077	44.87832	4.1458100E 02	-2.5531384E 02	2.8698730E 00
49	199.341	44.87818	4.1438575E 02	-2.5555253E 02	2.8698730E 00
50	199.605	44.87804	4.1419050E 02	-2.5579122E 02	2.8698730E 00
51	199.869	44.87790	4.1400000E 02	-2.5602991E 02	2.8698730E 00
52	200.133	44.87776	4.1380475E 02	-2.5626860E 02	2.8698730E 00
53	200.397	44.87762	4.1360950E 02	-2.5650729E 02	2.8698730E 00
54	200.661	44.87748	4.1341425E 02	-2.5674598E 02	2.8698730E 00



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*      WILSON,FLORENCE  PLATE CALIBRATION PROGRAM OF FEB. 18,1965
*      LIST
*      LABEL
*      SYMBOL TABLE

CPCAL      PLATE CALIBRATION PROGRAM
C
C      DIMENSION STATEMENTS
      DIMENSION XX(5),XY(5),XZ(5),YX(5),YY(5),YZ(5),ZX(5),ZY(5),ZZ(5),
      1X(25),Y(25),EL(25),EM(25),EN(25),COSSIG(25),EXI(25),ETA(25),TITLE(
      212)
C
C      COMMON STATEMENTS
      COMMON TITLE,EXI,ETA,X,Y,N
C
C      FORMAT STATEMENTS
20 FORMAT(12A6)
21 FORMAT(10I5)
22 FORMAT(72H
1
)
23 FORMAT(3E15.8)
24 FORMAT(1H 9F13.8)
25 FORMAT(2F4.0,F8.4,2F4.0,F7.3,2F10.5,15,E15.8)
26 FORMAT(17H0RT. ASC. CENTER=F4.0,2H HF4.0,2H MF8.4,2H S/13H DEC. CE
      INTER=F4.0,2H DF4.0,2H MF7.3,2H S/4H XC=F10.5,7H      YC=F10.5/8H DTH
      ZETA=1PE15.7)
27 FORMAT(72H0 RIGHT ASCENSION      DECLINATION      X
1      Y      /37H HR      MIN      SEC      DEG      MIN      SEC)
28 FORMAT(1H F4.0,F5.0,F9.4,F8.0,F5.0,F8.3,F13.3,F13.3)
29 FORMAT(17H0RT. ASC. CENTER=F4.0,2H HF4.0,2H MF8.4,2H S/13H DEC. CE
      INTER=F4.0,2H DF4.0,2H MF7.3,2H S/6H0LAMC=1PE15.7,10H      MUC=1PE1
      25.7,10H      NUC=1PE15.7/8H LAMETA=1PE15.7,10H      MUETA=1PE15.7,10
      3H      NUETA=1PE15.7/8H LAMEXI=1PE15.7,10H      MUEXI=1PE15.7,10H      N
      4UEXI=1PE15.7)
30 FORMAT(104H0NO.      LAMDA      MU      NU
1      COSSIG      ETA      XI)
31 FORMAT(14,1P6E18.7)
32 FORMAT(3H A=1PE15.7,10H      B=1PE15.7,10H      C=1PE15.7,10H
1      D=1PE15.7/6H ACAP=1PE15.7,10H      BCAP=1PE15.7,10H      CCAP
2=1PE15.7,10H      DCAP=1PE15.7/7H STARS 15,2H 15)
33 FORMAT(72H0NO      X(S)      Y(S)      DX      DY
1
)
34 FORMAT(13,2F10.3,2F10.5)
35 FORMAT(1H 1P3E15.7)
36 FORMAT(32H0CORRECTED COORDINATES OF CENTER)
37 FORMAT(6H0LAMC=1PE15.7,10H      MUC=1PE15.7,10H      NUC=1PE15.7/8
1H LAMETA=1PE15.7,10H      MUETA=1PE15.7,10H      NUETA=1PE15.7/8H LAME
2XI=1PE15.7,10H      MUEXI=1PE15.7,10H      NUEXI=1PE15.7)
38 FORMAT(6H0AEXI=1PE15.7,8H      BEXI=1PE15.7,8H      CEXI=1PE15.7/6H AETA
1=1PE15.7,8H      BETA=1PE15.7,8H      CETA=1PE15.7)
C
C      DEFINE PI AND DTRAD=DEGREES TO RADIAN
      PI=3.141592654
      DTRAD=3.141592654/180.
C
C      READ AND WRITE TITLE
1 READ INPUT TAPE 2,20,TITLE
WRITE OUTPUT TAPE 3,20,TITLE

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C      NUMBER OF SETS OF PRECESSION CONSTANTS AND NUMBER OF STARS
C      READ INPUT TAPE 2,21,NN,N
C
C      READ IN FOUR CARDS FOR EACH PRECESSION SET
C      CARD ONE GIVES DATES FOR PRECESSION
C      CARDS TWO-FOUR GIVE PRECESSION CONSTANTS--THREE PER CARD
C      DO 100 I=1,NN
C      READ INPUT TAPE 2,22
C      READ INPUT TAPE 2,23,XX(I),XY(I),XZ(I),YX(I),YY(I),YZ(I),ZX(I),ZY(
100 1),ZZ(I)
C      WRITE OUTPUT TAPE 3,22
C      100 WRITE OUTPUT TAPE 3,24,XX(I),XY(I),XZ(I),YX(I),YY(I),YZ(I),ZX(I),Z
1Y(I),ZZ(I)
C
C      PLATE CENTER CALCULATIONS
C
C      READ AND WRITE PLATE CENTER DATA
C      READ INPUT TAPE 2,25,ACHR,ACMIN,ACSEC,DCDEG,DCMIN,DCSEC,XC,YC,M,DT
1H
C      WRITE OUTPUT TAPE 3,26,ACHR,ACMIN,ACSEC,DCDEG,DCMIN,DCSEC,XC,YC,DT
1H
C      COMPUTE RIGHT ASCENSION AND DECLINATION OF PLATE CENTER
C      IN DEGREES AND RADIANS
C      AC=15.*(ACHR+(ACMIN+ACSEC/60.)/60.)
C      DC=DCDEG+(DCMIN+DCSEC/60.)/60.
C      AC=DTRAD*AC
C      DC=DTRAD*DC
C      COMPUTE UNPRECESSED DIRECTION COSINES OF PLATE CENTER
C      EQ. 14
C      IF(DC)60,61,62
C      60 DC=-DC
C      ENC=-SINF(DC)
C      GO TO 63
C      61 ENC=0.
C      GO TO 63
C      62 ENC=SINF(DC)
C      63 ELC=COSF(DC)*COSF(AC)
C      EMC=COSF(DC)*SINF(AC)
C
C      PRECESSION OF PLATE CENTER
C      M IS THE PRECESSION SET TO USE FOR PLATE CENTER
C      I=M
C      PRECESS PLATE CENTER DIRECTION COSINES BY EQ. 5
C      ELCP=XX(I)*ELC+XY(I)*EMC+XZ(I)*ENC
C      EMCP=YX(I)*ELC+YY(I)*EMC+YZ(I)*ENC
C      ENCP=ZX(I)*ELC+ZY(I)*EMC+ZZ(I)*ENC
C
C      COMPUTE DIRECTION COSINES OF XI AND ETA AXES--EQ. 15,16
C      ENETA=SQRTF(1.-ENCP*ENCP)
C      ELETA=-ENCP*ELCP/ENETA
C      EMETA=-ENCP*EMCP/ENETA
C      ELEXI=-EMCP/ENETA
C      EMEXI=ELCP/ENETA
C      ENEXI=0.
C
C      WRITE COLUMN HEADINGS
C      WRITE OUTPUT TAPE 3,27
C

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C          CALCULATIONS FOR STARS
C          PERFORM LOOP FOR EACH STAR
C          DO 101 J=1,N
C              READ IN DATA FOR CURRENT STAR
C              READ INPUT TAPE 2,25,AHR,AMIN,ASEC,DDEG,DMIN,DSEC,X(J),Y(J),M
C              WRITE OUTPUT TAPE 3,28,AHR,AMIN,ASEC,DDEG,DMIN,DSEC,X(J),Y(J)
C              COMPUTE UNPRECESSED RIGHT ASCENSION AND DECLINATION
C              OF STAR IN DEGREES AND RADIAN
C          65 A=15.*(AHR+(AMIN+ASEC/60.)/60.)
C          66 D=DDEG+(DMIN+DSEC/60.)/60.
C              A=DTRAD*A
C              D=DTRAD*D
C          COMPUTE UNPRECESSED DIRECTION COSINES OF STAR --EQ. 4
C          IF(D)67,68,69
C          67 D=-D
C              ENS=-SINF(D)
C              GO TO 70
C          68 ENS=0.
C              GO TO 70
C          69 ENS=SINF(D)
C          70 ELS=COSF(D)*COSF(A)
C              EMS=COSF(D)*SINF(A)
C
C          PRECESSION OF CURRENT STAR
C          M=PRECESSION SET TO USE WITH CURRENT STAR
C          I=M
C          PRECESS DIRECTION COSINES OF CURRENT STAR--EQ. 5
C          EL(J)=ELS*XX(I)+EMS*XY(I)+ENS*XZ(I)
C          EM(J)=ELS*YX(I)+EMS*YY(I)+ENS*YZ(I)
C          EN(J)=ELS*ZX(I)+EMS*ZY(I)+ENS*ZZ(I)
C
C          COMPUTE STANDARD COORDINATES OF CURRENT STAR--EQ. 18,19
C          COSSIG(J)=EL(J)*ELCP+EM(J)*EMCP+EN(J)*ENCP
C          EXI(J)=(EL(J)*ELEXI+EM(J)*EMEXI)/COSSIG(J)
C          101 ETA(J)=(EL(J)*ELETA+EM(J)*EMETA+EN(J)*ENETA)/COSSIG(J)
C
C          WRITE OUT DATA COMPUTED FOR PLATE CENTER
C          WRITE OUTPUT TAPE 3,20,TITLE
C          WRITE OUTPUT TAPE 3,29,ACHR,ACMIN,ACSEC,DCDEG,DCMIN,DCSEC,ELCP,EMC
C          1P,ENCP,ELETA,EMETA,ENETA,ELEXI,EMEXI,ENEXI
C          WRITE COLUMN HEADINGS
C          WRITE OUTPUT TAPE 3,30
C
C          WRITE OUT COMPUTED DIRECTION COSINES AND STANDARD
C          COORDINATES FOR EACH STAR
C          DO 102 J=1,N
C          102 WRITE OUTPUT TAPE 3,31,J,EL(J),EM(J),EN(J),COSSIG(J),ETA(J),EXI(J)
C
C          NUMBER OF FOUR CONSTANT SOLUTIONS TO PERFORM
C          READ INPUT TAPE 2,21,MM
C
C          LOOP FOR EACH FOUR CONSTANT SOLUTION
C          DO 103 J=1,MM
C          READ IN STARS TO USE FOR END POINTS IN SOLUTION
C          READ INPUT TAPE 2,21,L,LL
C
C          PERFORM FOUR CONSTANT SOLUTION
C          COMPUTE A,B,C,D FOR THESE TWO STARS--EQ. 20

```

```

DEN=(ETA(LL)-ETA(L))**2+(EXI(LL)-EXI(L))**2
IF(DEN-1.E-10)103,104,104
104 A=(X(LL)-X(L))*(EXI(LL)-EXI(L))-(Y(LL)-Y(L))*(ETA(LL)-ETA(L))
A=A/DEN
B=(EXI(LL)-EXI(L))*(Y(LL)-Y(L))+(ETA(LL)-ETA(L))*(X(LL)-X(L))
B=B/DEN
C=X(LL)-A*EXI(LL)-B*ETA(LL)
D=Y(LL)-B*EXI(LL)+A*ETA(LL)
DEN=A*A+B*B
C IF DEN TOO SMALL IGNORE THIS STAR
IF(DEN-1.E-10)103,105,105
C
C COMPUTE INVERSE PLATE CONSTANTS--EQ. 21
105 ACAP=A/DEN
BCAP=B/DEN
CCAP=-(A*C+B*D)/DEN
DCAP=(A*D-B*C)/DEN
ECAP=-ACAP
C
C WRITE OUT PLATE CONSTANTS AND INVERSE PLATE CONSTANTS
C FROM FOUR CONSTANT SOLUTION
WRITE OUTPUT TAPE 3,20,TITLE
WRITE OUTPUT TAPE 3,32,A,B,C,D,ACAP,BCAP,CCAP,DCAP,L,LL
WRITE OUTPUT TAPE 3,38,ACAP,BCAP,CCAP,BCAP,ECAP,DCAP
C
C WRITE COLUMN HEADINGS
WRITE OUTPUT TAPE 3,33
C
C WRITE INVERSE PLATE CONSTANTS ON CARDS FOR USE IN
C TRAJECTORY PROGRAM
WRITE OUTPUT TAPE 14,20,TITLE
WRITE OUTPUT TAPE 14,35,ACAP,BCAP,CCAP,BCAP,ECAP,DCAP
C
C COMPUTE RESIDUALS FOR EACH STAR FOR FOUR CONSTANT WORK
C
DO 106 I=1,N
C COMPUTE DX AND DY FOR CURRENT STAR--EQ. 23
DX=A*EXI(I)+B*ETA(I)+C-X(I)
DY=B*EXI(I)-A*ETA(I)+D-Y(I)
C WRITE OUT DX AND DY FOR CURRENT STAR
106 WRITE OUTPUT TAPE 3,34,I,X(I),Y(I),DX,DY
103 CONTINUE
C
C PERFORM A SIX CONSTANT (LEAST SQUARES) SOLUTION
CALL SIXCON
C
C COMPUTE CORRECTED DIRECTION COSINES OF PLATE CENTER
C AND AXES --EQ. 45
C DTH READ IN AS SEC OF TIME MUST BE CONVERTED TO RADIANS
DTH=DTH*PI/43200.
SF=SINF(DTH)
CF=COSF(DTH)
ELC=ELCP*CF-EMCP*SF
EMC=EMCP*CF+ELCP*SF
ELEX=ELEXI*CF-EMEXI*SF
EMEX=EMEXI*CF+ELEXI*SF
ELET=ELETA*CF-EMETA*SF
EMET=EMETA*CF+ELETA*SF

```

```

C
C      WRITE OUT CORRECTED DIRECTION COSINES
      WRITE OUTPUT TAPE 3,20,TITLE
      WRITE OUTPUT TAPE 3,36
      WRITE OUTPUT TAPE 3,37,ELC,EMC,ENCP,ELET,EMET,ENETA,ELEX,EMEX,ENEX
11
C      WRITE CORRECTED DIRECTION COSINES ON CARDS
121 WRITE OUTPUT TAPE 14,35,ELC,ELEX,ELET,EMC,EMEX,EMET,ENCP,ENETA
C      DO NEXT SET OF DATA IF ANY
      GO TO 1
      END

```



```

*      LIST
*      LABEL
*      SYMBOL TABLE
*      SUBROUTINE SIXCON

CSCOM      SUBROUTINE FOR SIX CONSTANT SOLUTION
C
C
C      DIMENSION STATEMENTS
      DIMENSION TITLE(12),EXI(25),ETA(25),X(25),Y(25),A(10,10),B(10,10),
      1TWO(10)
C
      COMMON TITLE,EXI,ETA,X,Y,N
C
C      FORMAT STATEMENTS
20 FORMAT(12A6)
21 FORMAT(6H0AEXI=1PE15.7,8H      BEXI=1PE15.7,8H      CEXI=1PE15.7/6H AETA
      1=1PE15.7,8H      BETA=1PE15.7,8H      CETA=1PE15.7)
22 FORMAT(4H0AX=1PE15.7,8H      BX=1PE15.7,8H      CX=1PE15.7/4H AY=1PE
      115.7,8H      BY=1PE15.7,8H      CY=1PE15.7)
23 FORMAT(1H 1P3E15.7/1H 1P3E15.7)
23 FORMAT(72H0NO      X(S)      Y(S)      DX      DY
      1
      )
34 FORMAT(13,2F10.3,2F10.5)
35 FORMAT(8HOMATRIX=15)
C
C      SET ALL SUMS TO ZERO AT START
      ONE=1.
      SEXI2=0.
      SEXIET=0.
      SEXI=0.
      SEXEXI=0.
      SETA2=0.
      SETA=0.
      SEXETA=0.
      SEX=0.
      SEYEXI=0.
      SEYETA=0.
      SEY=0.
      EN=N
C
C      N=NUMBER OF STARS
C      PERFORM LOOP FOR EACH STAR
      DO 100 J=1,N
C
C      SUM OF XI SQUARED
      SEXI2=SEXI2+EXI(J)*EXI(J)
C      SUM OF XI TIMES ETA
      SEXIET=SEXIET+EXI(J)*ETA(J)
C      SUM OF XI
      SEXI=SEXI+EXI(J)
C      SUM OF X TIMES XI
      SEXEXI=SEXEXI+X(J)*EXI(J)
C      SUM OF ETA SQUARED
      SETA2=SETA2+ETA(J)*ETA(J)
C      SUM OF ETA
      SETA=SETA+ETA(J)
C      SUM OF X TIMES ETA

```

```

SEXETA=SEXETA+X(J)*ETA(J)
C      SUM OF X
SEX=SEX+X(J)
C      SUM OF Y TIMES XI
SEYEXI=SEYEXI+Y(J)*EXI(J)
C      SUM OF Y TIMES ETA
SEYETA=SEYETA+Y(J)*ETA(J)
C      SUM OF Y
100 SEY=SEY+Y(J)
C
C      SET UP MATRICES AS IN EQ. 46
A(1,1)=SEX12
A(1,2)=SEX1ET
A(1,3)=SEX1
A(2,1)=SEX1ET
A(2,2)=SETA2
A(2,3)=SETA
A(3,1)=SEX1
A(3,2)=SETA
A(3,3)=EN
B(1,1)=SEXEXI
B(2,1)=SEXETA
B(3,1)=SEX
B(1,2)=SEYEXI
B(2,2)=SEYETA
200 B(3,2)=SEY
C
C      SOLVE MATRIX EQUATION BY LIBRARY ROUTINE XSIMEQF
MATRIX=XSIMEQF(10,3,2,A,B,ONE,TWO)
C
C      WRITE OUTPUT TAPE 3,20,TITLE
C      PRINT OUT CODE WORD=1 IF SOLUTION SUCCESSFUL
C      OTHERWISE IS GREATER THAN 1
WRITE OUTPUT TAPE 3,35,MATRIX
C
C      WRITE OUT PLATE CONSTANTS FROM SIX CONSTANT SOLUTION
WRITE OUTPUT TAPE 3,22,A(1,1),A(2,1),A(3,1),A(1,2),A(2,2),A(3,2)
C
C      COMPUTE INVERSE PLATE CONSTANTS--EQ. 23
CXY=A(1,1)*A(2,2)-A(1,2)*A(2,1)
ASEXI=A(2,2)/CXY
BSEXI=-A(2,1)/CXY
CSEXI=(-A(2,2)*A(3,1)+A(2,1)*A(3,2))/CXY
ASETA=-A(1,2)/CXY
BSETA=A(1,1)/CXY
CSETA=(A(1,2)*A(3,1)-A(1,1)*A(3,2))/CXY
C
C      WRITE OUT INVERSE PLATE CONSTANTS
WRITE OUTPUT TAPE 3,21,ASEXI,BSEXI,CSEXI,ASETA,BSETA,CSETA
C      PUT INVERSE PLATE CONSTANTS ON CARDS
WRITE OUTPUT TAPE 14,23,ASEXI,BSEXI,CSEXI,ASETA,BSETA,CSETA
C
C      WRITE OUT COLUMN HEADINGS
WRITE OUTPUT TAPE 3,33
C
C      COMPUTE AND WRITE OUT RESIDUALS FOR EACH STAR
C      COMPUTED BY SIX CONSTANT SOLUTION
DO 101 J=1,N

```

```
DX=A(1,1)*EXI(J)+A(2,1)*ETA(J)+A(3,1)-X(J)
DY=A(1,2)*EXI(J)+A(2,2)*ETA(J)+A(3,2)-Y(J)
101 WRITE OUTPUT TAPE 3,34,J,X(J),Y(J),DX,DY
C
C      RETURN TO MAIN PROGRAM
      RETURN
      END
```

```

* WILSON,FLORENCE OPTICAL TRAJECTORY ASSEMBLY OF 2/17/65
* LIST
* LABEL
* SYMBOL TABLE

```

```
CTRAJ OPTICAL TRAJECTORY REDUCTION PROGRAM
```

```

C
C DIMENSION STATEMENTS
DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
20,2),ELC(2),ELET(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)

```

```

C
C COMMON STATEMENTS
COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
1DLONG,DTRAD,EL,ELAZ,ELC,ELET,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
3P,ENR,ENZ,ETAAB,ETAABO,ETAP,EXIAB,EXIABO,EXIP,F2,GTHR,GTM,GTS,H,HE
4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
7TR,XO,Y,YBAR,YC,YP,YQ,YTR,YO,ZETAAB,ZETAP,ZTRAIL

```

```

C
C FORMAT STATEMENTS
20 FORMAT(12A6)
21 FORMAT(10I5)
22 FORMAT(3(2F4.0,F8.4),F11.7)
23 FORMAT(7H0THETO=F4.0,1HMF4.0,1HMF8.4,1HS)
24 FORMAT(6H TIME(11,2H)=F4.0,1HMF4.0,1HMF8.4,2HS /5H PHI(11,2H)=F4.0
1,1HDF4.0,1HMF8.4,1HS/6H LONG(11,2H)=F4.0,1HMF4.0,1HMF8.4,1HS/6H EL
2EV(11,2H)=F11.7,4H KFT)
25 FORMAT(4E15.8)
26 FORMAT(3E15.8)
27 FORMAT(7F10.5)
28 FORMAT(2I5,F10.5,2I5,F10.5,2I5,F10.5)
29 FORMAT(38H0ACCUMULATOR OVERFLOW AT END OF DASHPT)
30 FORMAT(30H0DIVIDE CHECK AT END OF DASHPT)
31 FORMAT(27H0NO CHECKS AT END OF DASHPT)
32 FORMAT(21H0ACCUMULATOR OVERFLOW)
33 FORMAT(13H0DIVIDE CHECK)
35 FORMAT(1H 4E15.8)
36 FORMAT(1H 3E15.8)
37 FORMAT(1H 7F10.5)
38 FORMAT(1H 2I5,F10.5,2I5,F10.5,2I5,F10.5)

```

```

C
C TURN OFF INDICATORS AT START FOR LATER TESTING
1 IF ACCUMULATOR OVERFLOW 10,10
10 IF DIVIDE CHECK 11,11
C

```

```

C      DEFINE PI AND DTRAD=DEGREES TO RADIAN
11 DTRAD=1.74532925E-2
   PI=3.141592654
C
C      READ AND WRITE TITLE
   READ INPUT TAPE 2,20,TITLE
   WRITE OUTPUT TAPE 3,20,TITLE
C      READ IN SIDEREAL TIME AT 0 HOURS Z FOR DATE OF EVENT
   READ INPUT TAPE 2,22,THETOH,THETOM,THETOS
   WRITE OUTPUT TAPE 3,23,THETOH,THETOM,THETOS
C
C      LOOP FOR READING IN DATA FROM EACH OF TWO STATIONS
   DO 100 J=1,2
   J=J
C      CURRENT STATION TITLE
   READ INPUT TAPE 2,20,(TITLE1(JL,J),JL=1,12)
   WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
C
C      DATA FOR COMPUTING RELATIVE COORDINATES
   GT=GIVEN TIME (GREENWICH)---PH=LATITUDE OF STATION
   ELON=LONGITUDE OF STATION---ELEV=ELEVATION OF STATION
C      READ INPUT TAPE 2,22,GTHR(J),GTM(J),GTS(J),PHD(J),PHM(J),PHS(J),EL
   IONH(J),ELONH(J),ELONM(J),ELEV(J)
   WRITE OUTPUT TAPE 3,24,J,GTHR(J),GTM(J),GTS(J),J,PHD(J),PHM(J),PHS
   1(J),J,ELONH(J),ELONM(J),ELONS(J),J,ELEV(J)
C
C      DATA FOR COMPUTING TRAIL EQUATION
   N=NUMBER OF POINTS ALONG TRAIL--X0,Y0=APPROXIMATE X,Y
   ALONG TRAIL---XTR,YTR=ADDITION TO X0,Y0 TO GIVE EXACT
   READING
C      READ INPUT TAPE 2,21,N
   READ INPUT TAPE 2,25,X0,Y0
   READ INPUT TAPE 2,25,(XTR(JJ),YTR(JJ),JJ=1,N)
   WRITE OUTPUT TAPE 3,35,X0,Y0
   WRITE OUTPUT TAPE 3,35,(XTR(JJ),YTR(JJ),JJ=1,N)
C      COMPUTE TRAIL EQUATION FOR STATION BY SUBROUTINE TRAIL
   CALL TRAIL
C
C      READ IN DATA TO USE IN COMPUTING POLE OF EVENT
   DATA INCLUDES INVERSE PLATE CONSTANTS AND DIRECTION
   COSINES OF PLATE CENTER AND COORDINATE AXES
   DATA OBTAINED FROM PLATE CALIBRATION PROGRAM
62 READ INPUT TAPE 2,26,CAX(J),CBX(J),CCX(J),CAY(J),CBY(J),CCY(J),ELC
   1(J),ELEXI(J),ELETA(J),EMC(J),EMEXI(J),EMETA(J),ENC(J),ENETA(J)
   WRITE OUTPUT TAPE 3,36,CAX(J),CBX(J),CCX(J),CAY(J),CBY(J),CCY(J),E
   1LC(J),ELEXI(J),ELETA(J),EMC(J),EMEXI(J),EMETA(J),ENC(J),ENETA(J)
C
C      READ IN X VALUES FOR FIVE POINTS OF SPECIAL INTEREST
   READ INPUT TAPE 2,27,(X(K,J),K=1,5)
   WRITE OUTPUT TAPE 3,37,(X(K,J),K=1,5)
C
C      READ IN DATA FOR EACH DASH
   NUM=NUMBER OF DASHES READ AT THIS STATION
   NDASH=NUMBER OF DASH---NWT=WEIGHT(QUALITY) OF THIS DASH
   XP=ACTUAL X READING OF THIS DASH
   THREE DASHES PER CARD
C      READ INPUT TAPE 2,21,NUM(J)
   NUM=NUM(J)

```



```

READ INPUT TAPE 2,28,(NDASH(K,J),NWT(K,J),XP(K,J),K=1,MUM)
WRITE OUTPUT TAPE 3,38,(NDASH(K,J),NWT(K,J),XP(K,J),K=1,MUM)

C
C      READ IN DATA USED TO COMPUTE RELATIVE TIME
C      XQ,YQ=OBSERVED COORDINATES OF CENTER OF SHUTTER ROTATION
C      XC,YC=COORDINATES OF PROJECTION CENTER
C      F2=FOCAL LENGTH OF CAMERA SQUARED
C      IFPLUS=+1 FOR COUNTERCLOCKWISE ROTATION---IFPLUS=-1 FOR
C      CLOCKWISE ROTATION---IFPLUS=0 FOR NON SUPER SCHMIDT
C      P=PERIOD OF REVOLUTION OF SHUTTER---OCCULT=NUMBER OF
C      OCCULTATIONS PER REVOLUTION
C      IFEXT=+1 IF CARDS ARE TO BE MADE---IFEXT=0 IF NO CARDS
C      ARE TO BE MADE
301 READ INPUT TAPE 2,25,XQ(J),YQ(J),XC(J),YC(J),F2(J),P(J),OCCULT(J)
WRITE OUTPUT TAPE 3,35,XQ(J),YQ(J),XC(J),YC(J),F2(J),P(J),OCCULT(J)
READ INPUT TAPE 2,21,IFEXT(J),IFPLUS(J)
WRITE OUTPUT TAPE 3,38,IFPLUS(J)
C      READ IN DATA FOR STATION 2 WHEN STATION 1 IS DONE
100 CONTINUE

C
C      DATA ALL IN COMPUTER---START CALCULATIONS
C
C      WRITE TITLE ON NEW PAGE
WRITE OUTPUT TAPE 3,20,TITLE

C
C      COMPUTE RELATIVE COORDINATES FOR THE TWO STATIONS
CALL RELCO

C
C      COMPUTE THE POLE OF THE EVENT FOR EACH STATION
DO 101 J=1,2
J=J
CALL POLE
101 CONTINUE

C
C      COMPUTE THE RADIANT
CALL RAD

C
C      PERFORM LOOP FOR EACH STATION
DO 102 J=1,2
J=J
C      COMPUTE RANGES AND HEIGHTS FOR THE FIVE POINTS OF
C      SPECIAL INTEREST FOR CURRENT STATION
CALL RANDH
C      COMPUTE RANGES,HEIGHTS,COSINE OF ZENITH DIRECTION AND
C      DISTANCE ALONG THE TRAIL FOR EACH DASH OF CURRENT STATION
CALL DASHPT
C      CHECK INDICATORS AT THIS POINT---IN CASE PROJECTION AND
C      SHUTTER ROTATION CENTERS VERY CLOSE WILL SET DIVIDE CHECK
C      IN NEXT SUBROUTINE---THIS WILL NOT HARM RESULTS
IF ACCUMULATOR OVERFLOW 12,13
12 WRITE OUTPUT TAPE 3,29
IF DIVIDE CHECK 14,15
14 WRITE OUTPUT TAPE 3,30
GO TO 15
13 IF DIVIDE CHECK 16,17
16 WRITE OUTPUT TAPE 3,30
GO TO 15
17 WRITE OUTPUT TAPE 3,31

```

```

C
C      COMPUTE RELATIVE TIMES FOR THE CURRENT STATION FOR EACH DASH
C      DIRECTION COSINES IN ALTITUDE AZIMUTH SYSTEM ALSO
C      COMPUTED FOR EACH POINT IN SUBROUTINE RELTIM
C      15 CALL RELTIM
C      COMPUTE FOR STATION 2 OR END PROGRAM HERE
102 CONTINUE
C
C      CHECK INDICATORS AT END OF PROGRAM
C      IF ACCUMULATOR OVERFLOW 200,201
200 WRITE OUTPUT TAPE 3,32
201 IF DIVIDE CHECK 202,203
202 WRITE OUTPUT TAPE 3,33
203 GO TO 1
      END

```

```
* LIST
* LABEL
* SYMBOL TABLE
* SUBROUTINE TRAIL
```

CTRAIL LEAST SQUARES LINE THROUGH TRAIL MAY 5, 1964

```
C
C      DIMENSION STATEMENTS
C      DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),COSZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(250,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(2560,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(29),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
C      DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
```

```
C
C      COMMON STATEMENTS
C      COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN3P,ENR,ENZ,ETAAB,ETAAB0,ETAP,EXIAB,EXIAB0,EXIP,F2,GTHR,GTM,GTS,H,HE4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X7TR,X0,Y,YBAR,YC,YP,YQ,YTR,Y0,ZETAAB,ZETAP,ZTRAIL
```

```
C
C      ERROR MESSAGE FORMAT
C      26 FORMAT(41HODENOMINATOR LESS THAN 1.E-10 IN TRAIL EQ)
```

```
C      START COMPUTATION
C      N=NUMBER OF POINTS ON TRAIL
C      EN=N
C      SET SUMS EQUAL TO ZERO AT START
C      SUMX=0.
C      SUMY=0.
C      SUMXY=0.
C      SUMX2=0.
```

```
C      COMPUTE SUMS OF POINTS ON TRAIL
C      DO 100 JJ=1,N
C      SUM OF X
C      SUMX=SUMX+XTR(JJ)
C      SUM OF Y
C      SUMY=SUMY+YTR(JJ)
C      SUM OF X TIMES Y
C      SUMXY=SUMXY+XTR(JJ)*YTR(JJ)
C      SUM OF X SQUARED
C      SUMX2=SUMX2+XTR(JJ)*XTR(JJ)
C      100 CONTINUE
```

```
C      COMPUTE AVERAGE X, AVERAGE Y AND SLOPE
C      XBAR(J)=SUMX/EN
```

```

      YBAR(J)=SUMY/EN
C      COMPUTE SLOPE---EQ. 26
      SLOPE(J)=SUMXY-EN*XBAR(J)*YBAR(J)
      EMD=SUMX2-EN*XBAR(J)**2
C      IF DENOMINATOR TOO SMALL WRITE ERROR MESSAGE
      IF(EMD-1.E-10)92,91,91
92  WRITE OUTPUT TAPE 3,26
91  SLOPE(J)=SLOPE(J)/EMD
C      VALUE OF AVERAGE X IN MM
      XBAR(J)=XBAR(J)+X0
C      VALUE OF AVERAGE Y IN MM
102 YBAR(J)=YBAR(J)+Y0
C      COMPUTE Y INTERCEPT---EQ. 25
      ZTRAIL(J)=YBAR(J)-SLOPE(J)*XBAR(J)
C      RETURN TO MAIN PROGRAM
      RETURN
      END

```

```

*      LIST
*      LABEL
*      SYMBOL TABLE
*      SUBROUTINE RELCO

C      RC RELATIVE COORDINATES
C
C      DIMENSION STATEMENTS
      DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
      DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
C
C      COMMON STATEMENTS
      COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
3P,ENR,ENZ,ETAAB,ETAAB0,ETAP,EXIAB,EXIAB0,EXIP,F2,GTHR,GTM,GTS,H,HE
4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
7TR,XO,Y,YBAR,YC,YP,YQ,YTR,YO,ZETAAB,ZETAP,ZTRAIL
C
C      FORMAT STATEMENTS
20 FORMAT(12A6)
24 FORMAT(16H TIME(11,2H)=F4.0,1HMF4.0,1HMF8.4,2HS /5H PHI(11,2H)=F4.0
1,1HDF4.0,1HMF8.4,1HS/6H LONG(11,2H)=F4.0,1HMF4.0,1HMF8.4,1HS/6H EL
2EV(11,2H)=F11.7,4H KFT)
26 FORMAT(17H THETA(11,2H)=1PE15.7,4H DEG/5H ELZ(11,2H)=1PE15.7,10H
1 EMZ(11,2H)=1PE15.7,10H ENZ(11,2H)=1PE15.7/10H PHIPRIME(11,
22H)=1PE15.7,4H DEG/5H RHO(11,2H)=1PE15.7,4H KFT)
27 FORMAT(18HOEXIAB0=1PE15.7,14H KFT ETAAB0=1PE15.7,4H KFT/7H EXIAB=
11PE15.7,13H KFT ETAAB=1PE15.7,14H KFT ZETAAB=1PE15.7,4H KFT/7H
2 ERROR=1PE15.7,4H KFT)
C
C      COMPUTE SIDEREAL TIMES AND GEOGRAPHIC COORDINATES
C      FOR CURRENT STATION
      DO 101 J=1,2
      J=J
C      WRITE STATION TITLE AND INPUT COORDINATES
      WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
      WRITE OUTPUT TAPE 3,24,J,GTHR(J),GTM(J),GTS(J),J,PHD(J),PHM(J),PHS
1(J),J,ELONH(J),ELONM(J),ELONS(J),J,ELEV(J)
C
C      COMPUTE UNIVERSAL TIME IN HOURS
      UT=GTHR(J)+(GTM(J)+GTS(J)/60.)/60.
C      COMPUTE CORRECTION IN THETA-- EQ. 9
      DTHETS=236.555*(UT/24.)
      DTHETS=DTHETS/3600.
C      COMPUTE THETA0 IN HOURS

```



```

      TTTHETO=THETOH+(THETOM+THETOS/60.)/60.
C      COMPUTE LONGITUDE OF STATION IN HOURS
      ELON(J)=ELONH(J)+(ELONM(J)+ELONS(J)/60.)/60.
C
C      COMPUTE SIDEREAL TIME IN DEGREES AND RADIANS--EQ. 10
      THET=15.*(TTTHETO+UT-ELON(J)+DTHETS)
      THETA(J)=THET*DTRAD
      CTH(J)=COSF(THETA(J))
      STH(J)=SINF(THETA(J))
C
C      LATITUDE CALCULATIONS
      PHI(J)=(PHD(J)+(PHM(J)+PHS(J)/60.)/60.)*DTRAD
      SPHI(J)=SINF(PHI(J))
      CPHI(J)=COSF(PHI(J))
C      COMPUTE DIRECTION COSINES OF ZENITH DIRECTION--EQ. 31
      ELZ(J)=CPHI(J)*CTH(J)
      EMZ(J)=CPHI(J)*STH(J)
      ENZ(J)=SPHI(J)
C      COMPUTE GEOCENTRIC LATITUDE OF STATION--EQ. 28
69 PHIP(J)=-695.6635*SINF(2.*PHI(J))+1.1731*SINF(4.*PHI(J))-0.0026*SIN
1F(6.*PHI(J))
70 PHIP(J)=PHS(J)+PHIP(J)
      PHIPD=PHD(J)+(PHM(J)+PHIP(J)/60.)/60.
      PHIP(J)=PHIPD*DTRAD
C      COMPUTE EARTHS RADIUS OF STATION--EQ. 28
71 RHO(J)=0.99832005+1.683494E-3*COSF(2.*PHI(J))-3.549E-6*COSF(4.*PHI
1(J))+8.E-9*COSF(6.*PHI(J))
72 RHO(J)=20.9264279E+3*RHO(J)
73 RHO(J)=RHO(J)+ELEV(J)
      CPHIP(J)=COSF(PHIP(J))
      SPHIP(J)=SINF(PHIP(J))
C      LONGITUDE CALCULATIONS
      ELON(J)=15.*(ELONH(J)+(ELONM(J)+ELONS(J)/60.)/60.)*DTRAD
C
C      WRITE OUT SIDEREAL TIME,DIRECTION COSINES OF ZENITH
C      DIRECTION,GEOCENTRIC LATITUDE AND EARTHS RADIUS
101 WRITE OUTPUT TAPE 3,26,J,THET,J,ELZ(J),J,EMZ(J),J,ENZ(J),J,PHIPD,J
1,RHO(J)
C
C      CALCULATE LONGITUDE DIFFERENCE BETWEEN STATIONS
C      LONGITUDE MEASURED POSITIVE TOWARD WEST
74 DL=ELON(1)-ELON(2)
C      IF VERY SMALL USE SERIES TO CALCULATE COS AND SIN
      IF(ABSF(DL-0.1))107,113,113
107 CDL=COSF(ELON(1))*COSF(ELON(2))+SINF(ELON(1))*SINF(ELON(2))
      SDL=SINF(ELON(1))*COSF(ELON(2))-COSF(ELON(1))*SINF(ELON(2))
      GO TO 114
113 IF(DL)108,109,110
108 DL=-DL
      SDL=-SINF(DL)
      GO TO 111
109 SDL=0.
      GO TO 111
110 SDL=SINF(DL)
111 CDL=COSF(DL)
C
C
C      CALCULATE DIRECTION COMPONENTS OF VECTOR FROM STATION 1

```

```

C          TO STATION 2 AT SIDEREAL TIME ZERO AT STATION 1--EQ. 29
114 EXIAB0=RHO(2)*CPHIP(2)*CDL-RHO(1)*CPHIP(1)
    ETAAB0=RHO(2)*CPHIP(2)*SDL
    ZETAAB=RHO(2)*SPHIP(2)-RHO(1)*SPHIP(1)
C          RANGE BETWEEN STATION 1 AND STATION 2--EQ. 29
    RAB=SQRTF(EXIAB0**2+ETAAB0**2+ZETAAB**2)
C
C          COMPUTE DIRECTION COMPONENTS OF VECTOR FROM STATION 1
C          TO STATION 2 AT TIME OF EVENT--EQ. 30
    EXIAB=EXIAB0*CTH(1)-ETAAB0*STH(1)
    ETAAB=ETAAB0*CTH(1)+EXIAB0*STH(1)
    ERROR=SQRTF(EXIAB**2+ETAAB**2+ZETAAB**2)-RAB
C
C          WRITE OUT DIRECTION COMPONENTS OF THESE VECTORS
112 WRITE OUTPUT TAPE 3,27,EXIAB0,ETAAB0,EXIAB,ETAAB,ZETAAB,ERROR
C          RETURN TO MAIN PROGRAM
    RETURN
    END

```

```

* LIST
* LABEL
* SYMBOL TABLE
* SUBROUTINE POLE

C POLE CALCULATION OF POLE OF TRAIL
C
C DIMENSION STATEMENTS
C DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
C DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
C
C COMMON STATEMENTS
C COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
3P,ENR,ENZ,ETAAB,ETAABO,ETAP,EXIAB,EXIABO,EXIP,F2,GTHR,GTM,GTS,H,HE
4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
7TR,XO,Y,YBAR,YC,YP,YQ,YTR,YO,ZETAAB,ZETAP,ZTRAIL
C
C FORMAT STATEMENTS
C 20 FORMAT(12A6)
29 FORMAT(5H0CAX=1PE15.7,7H CBX=1PE15.7,7H CCX=1PE15.7/5H CAY=1PE
115.7,7H CBY=1PE15.7,7H CCY=1PE15.7/5H ELC=1PE15.7,9H ELEXI=1
2PE15.7,9H ELETA=1PE15.7/5H EMC=1PE15.7,9H EMEXI=1PE15.7,9H E
3META=1PE15.7/5H ENC=1PE15.7,9H ENETA=1PE15.7)
31 FORMAT(3H0Y=1PE15.7,5H X +1PE15.7)
32 FORMAT(98H0 X Y EL
1EM EN ERROR)
33 FORMAT(1H F10.5,F15.5,1P4E20.7)
34 FORMAT(19H SINL LESS THAN E-5)
35 FORMAT(5H0ELP=1PE15.7,10H EMP=1PE15.7,10H ENP=1PE15.7/6H
1 SINL=1PE15.7/14H TRAIL LENGTH=1PE15.7,4H DEG/7HOERROR=1PE15.7,3H
2 1PE15.7,3H 1PE15.7)
C
C WRITE OUT TITLE, INVERSE PLATE CONSTANTS, AND DIRECTION
C COSINES OF PLATE CENTER AND COORDINATE AXES
C WRITE OUTPUT TAPE 3,20,TITLE
WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
63 WRITE OUTPUT TAPE 3,29,CAX(J),CBX(J),CCX(J),CAY(J),CBY(J),CCY(J),E
1LC(J),ELEXI(J),ELETA(J),EMC(J),EMEXI(J),EMETA(J),ENC(J),ENETA(J)
C WRITE OUT TRAIL EQUATION
WRITE OUTPUT TAPE 3,31,SLOPE(J),ZTRAIL(J)
C WRITE HEADS OF COLUMNS
WRITE OUTPUT TAPE 3,32
C
C CALCULATION OF STANDARD COORDINATES AND DIRECTION

```

```

C      COSINES FOR THE FIVE POINTS
C      PERFORM LOOP FOR THE FIVE POINTS
C      J=NUMBER OF STATION(I.E. 1 OR 2)---K=NUMBER OF POINT
C      READ AT THIS STATION
      DO 201 K=1,5
C      COMPUTE Y---EQ. 25
      Y(K,J)=SLOPE(J)*X(K,J)+ZTRAIL(J)
C      COMPUTE XIBAR AND ETABAR FOR GIVEN POINT---EQ. 23
      EXIBAR=CAX(J)*X(K,J)+CBX(J)*Y(K,J)+CCX(J)
      ETABAR=CAY(J)*X(K,J)+CBY(J)*Y(K,J)+CCY(J)
C      COMPUTE DIRECTION COSINES OF THIS POINT---EQ. 24
      DENOM=SQRTF(1.+EXIBAR**2+ETABAR**2)
      EL(K,J)=(ELC(J)+ELETA(J)*ETABAR+ELEXI(J)*EXIBAR)/DENOM
      EM(K,J)=(EMC(J)+EMETA(J)*ETABAR+EMEXI(J)*EXIBAR)/DENOM
      EN(K,J)=(ENC(J)+ENETA(J)*ETABAR)/DENOM
C      SUM OF DIRECTION COSINES SQUARED MUST BE ONE
      ERR1=EL(K,J)**2+EM(K,J)**2+EN(K,J)**2-1.
C
C      WRITE OUT X,Y AND DIRECTION COSINES OF THE POINT
201 WRITE OUTPUT TAPE 3,33,X(K,J),Y(K,J),EL(K,J),EM(K,J),EN(K,J),ERR1
C
C      COMPUTE POLE OF TRAIL---EQ. 27
      C1=EM(1,J)*EN(5,J)-EN(1,J)*EM(5,J)
      C2=EN(1,J)*EL(5,J)-EL(1,J)*EN(5,J)
      C3=EL(1,J)*EM(5,J)-EM(1,J)*EL(5,J)
      SINL(J)=SQRTF(C1**2+C2**2+C3**2)
      ASINL=ABSF(SINL(J))
      IF(ASINL-1.E-5)202,203,203
202 WRITE OUTPUT TAPE 3,34
203 IF(ASINL-1.)204,204,205
205 SINL(J)=1.
204 ELP(J)=C1/SINL(J)
      EMP(J)=C2/SINL(J)
      ENP(J)=C3/SINL(J)
C      COMPUTE ERRORS
      ERRP=ELP(J)**2+EMP(J)**2+ENP(J)**2-1.
      ERRP1=ELP(J)*EL(1,J)+EMP(J)*EM(1,J)+ENP(J)*EN(1,J)
      ERRP2=ELP(J)*EL(5,J)+EMP(J)*EM(5,J)+ENP(J)*EN(5,J)
C      COMPUTE TRAIL LENGTH
      DLONG(J)=ASINF(SINL(J))/DTRAD
C
C      WRITE OUT DIRECTION COSINES OF POLE,SIN TRAIL LENGTH
C      AND LENGTH OF TRAIL
      WRITE OUTPUT TAPE 3,35,ELP(J),EMP(J),ENP(J),SINL(J),DLONG(J),ERRP,
1ERRP1,ERRP2
C      RETURN TO MAIN PROGRAM
      RETURN
      END

```

```
* LIST
* LABEL
* SYMBOL TABLE
* SUBROUTINE RAD
```

```
C RAD CALCULATION OF RADIANT
```

```
C
C
```

#### DIMENSION STATEMENTS

```
DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
```

```
C
C
```

#### COMMON STATEMENTS

```
COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
3P,ENR,ENZ,ETAAB,ETAAB0,ETAP,EXIAB,EXIAB0,EXIP,F2,GTHR,GTM,GTS,H,HE
4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
7TR,XO,Y,YBAR,YC,YP,YQ,YTR,YO,ZETAAB,ZETAP,ZTRAIL
```

```
C
C
```

#### FORMAT STATEMENTS

```
20 FORMAT(12A6)
37 FORMAT(8HORADIAN)
38 FORMAT(5H0ELR=1PE15.7,7H EMR=1PE15.7,7H ENR=1PE15.7/6H SINQ=1P
1E15.7/7H ERROR=1PE15.7)
```

```
C
C
```

#### WRITE OUT TITLE

```
WRITE OUTPUT TAPE 3,20,TITLE
```

```
C
C
```

#### COMPUTE RADIANT---EQ. 32

```
C4=EMP(1)*ENP(2)-ENP(1)*EMP(2)
C5=ENP(1)*ELP(2)-ELP(1)*ENP(2)
C6=ELP(1)*EMP(2)-EMP(1)*ELP(2)
```

```
C
```

#### COMPUTE SIN Q---EQ. 32

```
SINQ=SQRTF(C4**2+C5**2+C6**2)
ELR=C4/SINQ
EMR=C5/SINQ
```

```
50 ENR=C6/SINQ
```

```
ERROR=ELR**2+EMR**2+ENR**2-1.+ELR*ELP(1)+EMR*EMP(1)+ENR*ENP(1)+ELR
1*ELP(2)+EMR*EMP(2)+ENR*ENP(2)
```

```
C
C
```

#### IS RADIANT ABOVE THE HORIZON

```
IF(ELZ(1)*ELR+EMZ(1)*EMR+ENZ(1)*ENR)7,8,8
```

```
C
```

```
NO, CHANGE SIGNS SO THAT IT IS ABOVE HORIZON
```

```
7 ELR=-ELR
EMR=-EMR
ENR=-ENR
```



C 8 WRITE OUTPUT TAPE 3,37  
C     WRITE OUT DIRECTION COSINES OF RADIANT AND SIN Q  
C 9 WRITE OUTPUT TAPE 3,38,ELR,EMR,ENR,SINQ,ERROR  
C     RETURN TO MAIN PROGRAM  
   RETURN  
   END

```
* LIST
* LABEL
* SYMBOL TABLE
* SUBROUTINE RANDH
```

CRANDH RANGES AND HEIGHTS FOR THE FIVE POINTS

```
C
C      DIMENSION STATEMENTS
      DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),COSZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(250,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),ELONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(2560,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
      DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
```

```
C
C      COMMON STATEMENTS
      COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN3P,ENR,ENZ,ETAAB,ETAABO,ETAP,EXIAB,EXIABO,EXIP,F2,GTHR,GTM,GTS,H,HEIGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X7TR,XO,Y,YBAR,YC,YP,YQ,YTR,YO,ZETAAB,ZETAP,ZTRAIL
```

```
C      FORMAT STATEMENTS
20 FORMAT(12A6)
33 FORMAT(1H F10.5,F15.5,1P3E20.7)
41 FORMAT(77H0      X      Y      R
      1H      DH)
```

```
C      WRITE OUT TITLES AND COLUMN HEADINGS
      WRITE OUTPUT TAPE 3,20,TITLE
      WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
62 WRITE OUTPUT TAPE 3,41
```

```
C      COMPUTE DISTANCE FROM STATION TO PLANE OF EVENT
      IS THIS STATION 1
      IF(J-1)206,207,206
      NO COMPUTE DISTANCE FROM STATION 2 TO PLANE DETERMINED
      BY TRAIL OF EVENT AND STATION 1---EQ. 33
206 L=1
      SST=-(EXIAB*ELP(L)+ETAAB*EMP(L)+ZETAAB*ENP(L))
      GO TO 208
C      YES COMPUTE DISTANCE FROM STATION 1 TO PLANE DETERMINED
C      BY TRAIL OF EVENT AND STATION 2---EQ. 33
207 L=2
      SST=EXIAB*ELP(L)+ETAAB*EMP(L)+ZETAAB*ENP(L)
```

```
C      COMPUTE RANGES, HEIGHTS AND COSINE OF ZENITH FOR EACH
C      POINT
C      J=NUMBER OF STATION--K=NUMBER OF POINT AT STATION
```

```

208 DO 205 K=1,5
C      COMPUTE COS EPSILON---EQ. 34
      SK=EL(K,J)*ELP(L)+EM(K,J)*EMP(L)+EN(K,J)*ENP(L)
C      COMPUTE RANGE OF POINT FROM STATION---EQ. 34
      R(K,J)=SST/SK
C      COMPUTE COSINE OF ZENITH DISTANCE OF POINT---EQ. 35
      COSZ1(K,J)=ELZ(J)*EL(K,J)+EMZ(J)*EM(K,J)+ENZ(J)*EN(K,J)
C      COMPUTE HEIGHT ABOVE SEA LEVEL OF POINT---EQ. 38
      H1=R(K,J)*COSZ1(K,J)
210 DH=SQRTF(RHO(J)*RHO(J)+R(K,J)*R(K,J)+2.*RHO(J)*H1)-(RHO(J)+H1)
      H(K,J)=H1+DH+ELEV(J)
C
C      WRITE OUT RANGE, HEIGHT, AND DH FOR EACH POINT
205 WRITE OUTPUT TAPE 3,33,X(K,J),Y(K,J),R(K,J),H(K,J),DH
C
C      RETURN TO MAIN PROGRAM
      RETURN
      END

```

```
* LIST
* LABEL
* SYMBOL TABLE
* SUBROUTINE DASHPT
```

CDASHPT RANGES AND HEIGHTS FOR THE DASHES

```
C
C      DIMENSION STATEMENTS
C      DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
C      DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
```

```
C
C      COMMON STATEMENTS
C      COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
3P,ENR,ENZ,ETAAB,ETAAB0,ETAP,EXIAB,EXIAB0,EXIP,F2,GTHR,GTM,GTS,H,HE
4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
7TR,XO,Y,YBAR,YC,YP,YQ,YTR,YO,ZETAAB,ZETAP,ZTRAIL
```

```
C
C      FORMAT STATEMENTS
C      20 FORMAT(12A6)
C      24 FORMAT(1H 15,2F15.5,1P4E20.7)
C      25 FORMAT(106H0 DASH          X          Y          D)          R
C      1          COSZ          H          D)
```

```
C
C      MUM=NUMBER OF POINTS OR DASHES TO PROCESS
C      MUM=NUM(J)
C      WRITE OUT TITLE AND COLUMN HEADINGS
C      WRITE OUTPUT TAPE 3,20,TITLE
C      WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
C      62 WRITE OUTPUT TAPE 3,25
C
C      IS THIS STATION 1
C      IF(J-1)102,103,102
C      NO COMPUTE DISTANCE FROM STATION 2 TO PLANE DETERMINED
C      BY TRAIL OF EVENT AND STATION 1--EQ. 33
C      102 L=1
C      SST=-(EXIAB*ELP(L)+ETAAB*EMP(L)+ZETAAB*ENP(L))
C      GO TO 104
C      YES COMPUTE DISTANCE FROM STATION 1 TO PLANE DETERMINED
C      BY TRAIL OF EVENT AND STATION 2--EQ. 33
C      103 L=2
C      SST=EXIAB*ELP(L)+ETAAB*EMP(L)+ZETAAB*ENP(L)
C      SET LINE COUNT TO 1
C      104 M=1
C
```

```

C          COMPUTE RANGE, HEIGHT, DIRECTION COSINES AND DISTANCES
C          ALONG THE TRAIL FOR EACH DASH
C          J=NUMBER OF STATION---K=NUMBER OF POINT AT STATION
DO 101 K=1,MUM
C          COMPUTE Y---EQ. 25
YP(K,J)=SLOPE(J)*XP(K,J)+ZTRAIL(J)
C          COMPUTE ETABAR AND XIBAR---EQ. 23
EXIBAR=CAX(J)*XP(K,J)+CBX(J)*YP(K,J)+CCX(J)
ETABAR=CAY(J)*XP(K,J)+CBY(J)*YP(K,J)+CCY(J)
C          COMPUTE DIRECTION COSINES---EQ. 24
DENOM=SQRTF(1.+EXIBAR**2+ETABAR**2)
PEL(K,J)=(ELC(J)+ELET(A(J)*ETABAR+ELEXI(J)*EXIBAR)/DENOM
PEM(K,J)=(EMC(J)+EMETA(J)*ETABAR+EMEXI(J)*EXIBAR)/DENOM
PEN(K,J)=(ENC(J)+ENETA(J)*ETABAR)/DENOM
C          COMPUTE COS EPSILON---EQ. 34
SK=PEL(K,J)*ELP(L)+PEM(K,J)*EMP(L)+PEN(K,J)*ENP(L)
C          COMPUTE RANGE FROM STATION---EQ. 34
RANGE(K,J)=SST/SK
C          COMPUTE ZENITH DIRECTION COSINE---EQ. 35
COSZ(K,J)=ELZ(J)*PEL(K,J)+EMZ(J)*PEM(K,J)+ENZ(J)*PEN(K,J)
C          COMPUTE HEIGHT ABOVE SEA LEVEL---EQ. 38
H1=RANGE(K,J)*COSZ(K,J)
DH=SQRTF(RHO(J)*RHO(J)+RANGE(K,J)*RANGE(K,J)+2.*RHO(J)*H1)-(RHO(J)
1+H1)
HEIGHT(K,J)=H1+DH+ELEV(J)
C          COMPUTE DIRECTION COMPONENTS OF POINT---EQ. 39
EXIP(K,J)=RANGE(K,J)*PEL(K,J)
ETAP(K,J)=RANGE(K,J)*PEM(K,J)
ZETAP(K,J)=RANGE(K,J)*PEN(K,J)
111 DD1=(EXIP(K,J)-EXIP(1,J))**2+(ETAP(K,J)-ETAP(1,J))**2+(ZETAP(K,J)-
1ZETAP(1,J))**2
DD1=SQRTF(DD1)
C
C          COMPUTE DISTANCE OF POINT FROM FIRST POINT ON TRAIL---EQ. 40
C          IS THIS THE FIRST POINT
C          IF(K-1)105,106,105
C          YES, DISTANCE OF FIRST POINT
106 D1=DD1
D(K,J)=D1
GO TO 107
C          NO COMPUTE DISTANCE FROM FIRST POINT
105 D(K,J)=DD1-D1
C
C          WRITE OUT DASH NUMBER, X, Y, RANGE, COSINE ZENITH DIRECTION,
C          HEIGHT, AND DISTANCE ALONG THE TRAIL
107 WRITE OUTPUT TAPE 3,24,NDASH(K,J),XP(K,J),YP(K,J),RANGE(K,J),COSZ(
1K,J),HEIGHT(K,J),D(K,J)
C          INCREMENT LINE COUNT
M=M+1
C          IS PAGE FINISHED
IF(M-50)101,101,108
C          YES START NEW PAGE WITH PROPER HEADINGS
108 WRITE OUTPUT TAPE 3,20,TITLE
WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
WRITE OUTPUT TAPE 3,25
C          SET LINE COUNT TO 1 FOR NEW PAGE
M=1
C          NO GO ON TO NEXT POINT

```



```
101 CONTINUE
C
C      RETURN TO MAIN PROGRAM
      RETURN
      END
```

```
* LIST
* LABEL
* SYMBOL TABLE
* SUBROUTINE RELTIM
```

CTIME CALCULATION OF TIME FOR THE DASHES

```
C
C      DIMENSION STATEMENTS
      DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
20,2),ELC(2),ELET(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
9),STH(2),THETA(2),TIME(250,2),TITLE(2),TITLE1(12,2)
      DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
```

```
C
C      COMMON STATEMENTS
      COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
1DLONG,DTRAD,EL,ELAZ,ELC,ELET,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
3P,ENR,ENZ,ETAAB,ETAABO,ETAP,EXIAB,EXIABO,EXIP,F2,GTHR,GTM,GTS,H,HE
4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
7TR,XO,Y,YBAR,YC,YP,YQ,YTR,YO,ZETAAB,ZETAP,ZTRAIL
```

```
C
C      FORMAT STATEMENTS
20 FORMAT(12A6)
21 FORMAT(4HOXQ=1PE15.7,7H      YQ=1PE15.7,7H      XC=1PE15.7,7H      YC=1P
1E15.7/4H F2=1PE15.7,7H      P=1PE15.7,11H      OCCULT=1PE15.7)
22 FORMAT(127HO DASH      X      TIME      RANGE
1      HEIGHT      ALPHA      BETA
2 GAMMA)
23 FORMAT(1H 15,F10.3,1P6E19.7)
24 FORMAT(22HODIVIDE CHECK AT DY/DX)
26 FORMAT(15,I3,2F10.6,F9.3,F8.3,3F9.6)
```

```
C
C      USE LONG FORM WHEN PROJECTION AND SHUTTER ROTATION
C      CENTERS MORE THAN A CM APART
C      LONG FORM FOR CALCULATING OMEGA---EQ. 44
      OMEGF(XMEAS)=ATANF(Z*(W*(XMEAS-XQ1)-DYTR)/(W*DYTR+(XMEAS-XQ1)))
C      SHORT FORM FOR COMPUTING OMEGA---EQ. 42
      SOMEGF(XMEAS)=ATANF((XMEAS-XQ1)/DYTR)
C      COMPUTE DT BY EQ. 43
      DTIF(OMEGA)=0.15915494*SIGN*(OMEGA-OMEGA0)*PER
C
C      CALCULATE CONSTANTS
      CON=P(J)/OCCULT(J)
      TWOPI=2.*PI
C
      MUM=NUM(J)
C      REDEFINITIONS OF QUANTITIES TO USE WITH FUNCTIONS
      XQ1=XQ(J)
```

```

      PER=P(J)
C      DISTANCE BETWEEN CENTER OF ROTATION AND PROJECTION
      DX=XQ(J)-XC(J)
      DY=YQ(J)-YC(J)
      W=DY/DX
C
C      MAY GIVE DIVIDE CHECK IF DX VERY SMALL--NEED NOT AFFECT
C      RESULTS IF SOMEGF USED
      IF DIVIDE CHECK 6,7
6 WRITE OUTPUT TAPE 3,24
7 D2=DX**2+DY**2
      Z=SQRTF(1.+D2/F2(J))
C
C      CALCULATION OF OMEGA0
C      DISTANCE OF POINT FROM Y OF ROTATION CENTER
      DYTR=YP(1,J)-YQ(J)
C      X COORDINATE OF THE FIRST POINT
      XMEAS=XP(1,J)
C      NEED WE USE THE LONG FORMULA
      IF(D2-100.)10,10,11
C      NO SET UP TO USE SHORT FORMULA
10 ASSIGN 50 TO KO
      ASSIGN 51 TO MO
      GO TO 70
C      YES SET UP TO USE LONG FORMULA
11 ASSIGN 60 TO KO
      ASSIGN 61 TO MO
70 GO TO KO,(50,60)
C      COMPUTE OMEGA0 BY SHORT FORMULA---EQ. 42
50 OMEGA0=SOMEGF(XMEAS)
C      SET CODE FOR SHORT FORMULA USE
      AA1=1.
      GO TO 71
C      COMPUTE OMEGA0 BY LONG FORMULA---EQ. 44
60 OMEGA0=OMEGF(XMEAS)
C      SET CODE FOR LONG FORMULA USE
      AA1=2.
C
C      LOOP FOR EACH POINT
C      J=NUMBER OF STATION---K=NUMBER OF POINT AT STATION
71 DO 100 K=1,MUM
C      COMPUTE DIRECTION COSINES OF POINT IN ALTITUDE AZIMUTH
C      SYSTEM---EQ. 12
      ELAZ(K,J)=PEM(K,J)*CTH(J)-PEL(K,J)*STH(J)
      EMAZ(K,J)=PEN(K,J)*CPHI(J)-(PEL(K,J)*CTH(J)+PEM(K,J)*STH(J))*SPHI(
1J)
      ENAZ(K,J)=(PEL(K,J)*CTH(J)+PEM(K,J)*STH(J))*CPHI(J)+PEN(K,J)*SPHI(
1J)
C      X MEASUREMENT OF POINT
      XMEAS=XP(K,J)
C      DISTANCE OF Y TO PROJECTION CENTER
      DYTR=YP(K,J)-YQ(J)
C      COMPUTE OMEGA OF POINT BY PROPER FORMULA
      GO TO MO,(51,61)
51 OMEGA=SOMEGF(XMEAS)
      AA2=1.
      GO TO 72

```

```

61 OMEGA=OMEGF(XMEAS)
   AA2=2.
C      SIGN OF ROTATION DIRECTION
72 SIGN=IFPLUS(J)
C      COMPUTE DT FOR POINT---EQ. 43
   DT=DTIF(OMEGA)
C      DASH NUMBER
75 ENDASH=NDASH(K,J)
C      COMPUTE RELATIVE TIME OF POINT---EQ. 41
100 TIME(K,J)=CON*ENDASH+DT
C
C      WRITE OUT TITLE AND CAMERA PARAMETERS AND HEADINGS
   WRITE OUTPUT TAPE 3,20,TITLE
   WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
   WRITE OUTPUT TAPE 3,21,XQ(J),YQ(J),XC(J),YC(J),F2(J),P(J),OCCULT(J)
103 WRITE OUTPUT TAPE 3,22
C      SET LINE COUNT TO 1 AT START OF PAGE
   M=1
C
C      WRITE OUT DASH NUMBER,X READING,RELATIVE TIME,RANGE,HEIGHT
C      AND DIRECTION COSINES ALTITUDE AZIMUTH SYSTEM EACH POINT
   DO 104 K=1,MUM
   WRITE OUTPUT TAPE 3,23,NDASH(K,J),XP(K,J),TIME(K,J),RANGE(K,J),HEI
1    GHT(K,J),ELAZ(K,J),EMAZ(K,J),ENAZ(K,J)
C      INCREMENT LINE COUNT
   M=M+1
C      END OF THIS PAGE
   IF(M-50)104,104,106
C      YES SET UP NEXT PAGE
106 WRITE OUTPUT TAPE 3,20,TITLE
   WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
   WRITE OUTPUT TAPE 3,22
C      SET LINE COUNT TO 1
   M=1
C      NO GO ON TO NEXT POINT
104 CONTINUE
C
C      DO WE WANT CARDS
   IF(IFEXT(J))200,201,200
C      YES WRITE DISTANCE,TIME,RANGE,HEIGHT,DIRECTION COSINES
C      ,DASH NUMBER AND DASH WEIGHT FOR EACH POINT ON CARDS
200 WRITE OUTPUT TAPE 14,20,TITLE
   WRITE OUTPUT TAPE 14,20,(TITLE1(JL,J),JL=1,12)
   DO 204 K=1,MUM
   WRITE OUTPUT TAPE 14,26,NDASH(K,J),NWT(K,J),D(K,J),TIME(K,J),RANGE
1    (K,J),HEIGHT(K,J),ELAZ(K,J),EMAZ(K,J),ENAZ(K,J)
204 CONTINUE
C      RETURN TO MAIN PROGRAM
201 RETURN
   END

```

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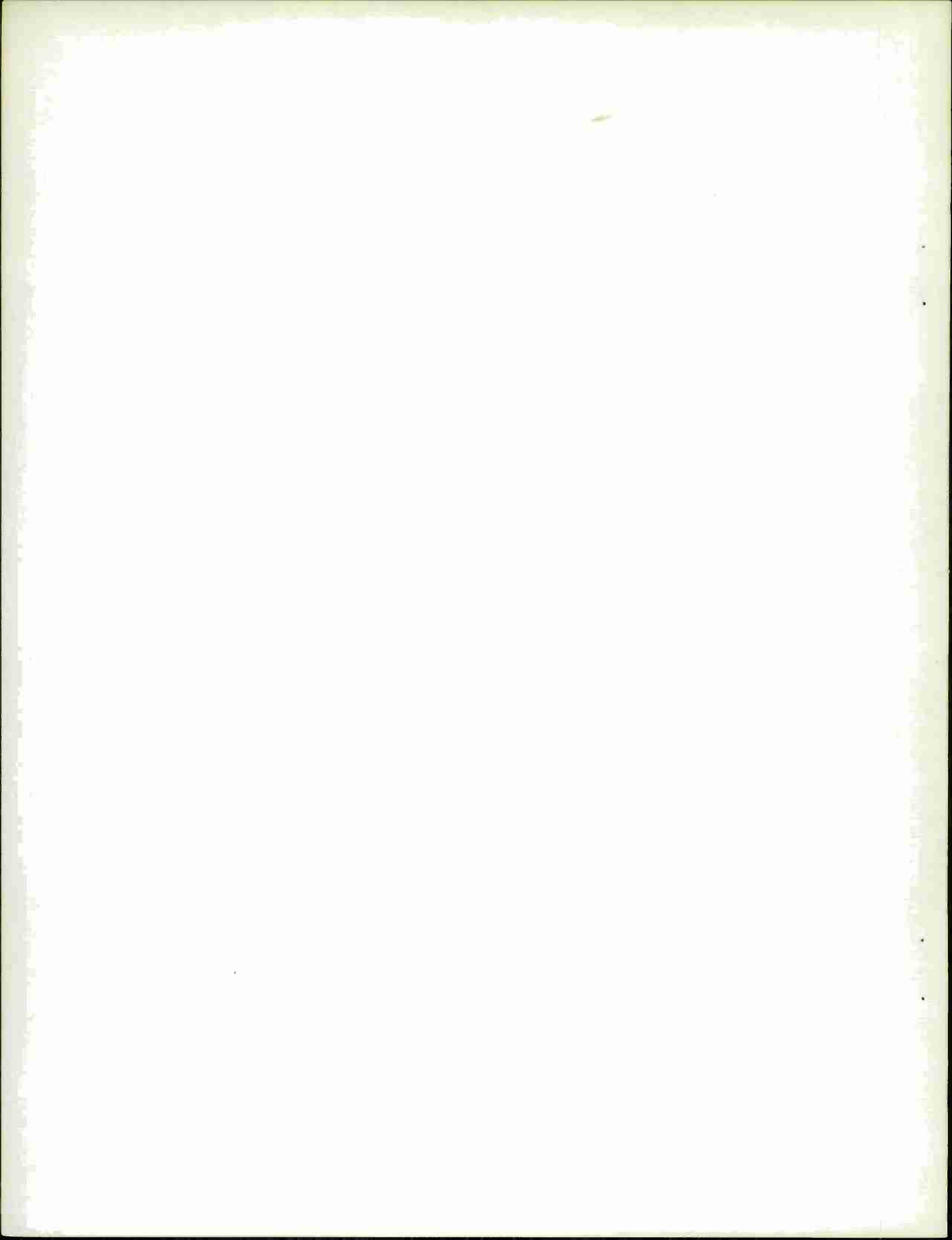
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